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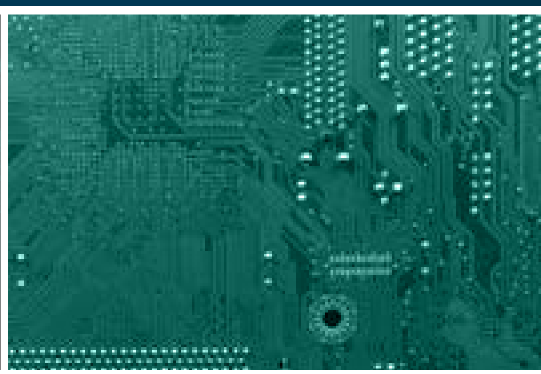
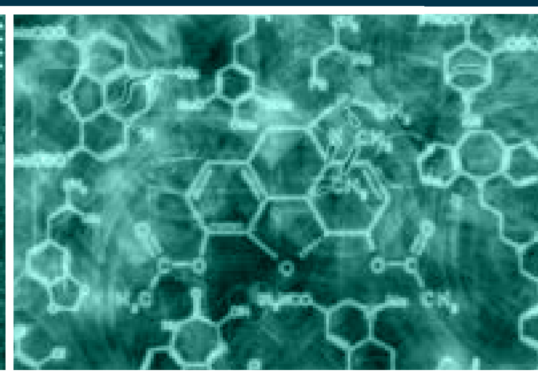

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INDEX OF CONTENT

4-BIT RIPPLE CARRY ADDER USING PROPOSED ENERGY EFFICIENT SINGLE PHASE ADIABATIC LOGIC TECHNIQUE

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ABSTRACT

This paper presents four bit ripple carry adder circuits based on low power adiabatic logic technique. The paper proposes a new design approach which is being derived from CMOS. A simulative investigation on the proposed circuit has been carried out in NI Multisim at 0.5µm CMOS technology with L=0.5µm and W=1.25µm. The power consumption is compared with conventional CMOS and a popular standard 2PASCL technique which shows great improvement in power dissipations.

Index Terms - Adiabatic logic, single phase, CMOS, energy recovery. Odd parity.

I. NTRODUCTION

“Adiabatic” is a Greek word and used to describe the thermodynamic processes, which means no energy exchange with environment (i.e., no entropy enters or leaves the system) and therefore dissipated energy is almost zero. Adiabatic Switching is commonly used to minimize the energy loss during charging/dis-charging process. This is accomplished by using time-varying voltage source instead of fixed voltage supply. Trapezoidal or AC power supplies is used to initially charge the circuit during specific adiabatic phases and then discharge the circuit to recover the supplied charge. The principle of adiabatic switching can be best explained by contrasting it with the conventional dissipative switching technique.[1]

With the widespread use of mobile and wireless devices and the increase of clock and logic speeds in meeting the new performance requirements, energy efficiency has become a key design aspect in the field of integrated circuits (ICs) [1]. Hence, adiabatic logic circuit is a new promising approach, which has been originally developed for low power digital circuits [2].

In this paper, we compared the power consumption of the newly proposed adiabatic circuit with the counterpart conventional CMOS and a pre-existing popular adiabatic family known as two phase clocked static CMOS logic 2PASCL [3]-[6]. The reference technique 2PASCL have good improvement in power consumption compared to other families such as QSERL, 2PADCL, ADCL, 1n1pSLN, 1n1p quqsi [4]. The proposed circuits show best energy saving. Comparison has shown a significant power saving to the extent of in case of proposed techniques as compared to CMOS logic within to transition frequency range..

II. CMOS THEORY

Power dissipation in conventional *CMOS* circuits primarily occurs during device switching. In conventional *CMOS* logic circuits (Fig.1), if an input is changed from 1 to 0 logic, the energy is transferred from the power supply to the output capacitor, the total charge $Q=C_L V_{DD}$ is supply to the output node and the energy which is being drawn from the power supply is $C_L V_{DD}^2$. But when the transition has ended, only half of the total energy is seen at the output load capacitor which is $C_L V_{DD}^2/2$ and the other half is lost in *PMOS* networks *(F)*. From V_{DD} to transition of the output node, energy stored in the load capacitance is dissipated in the *NMOS* network *(/F)* [7].

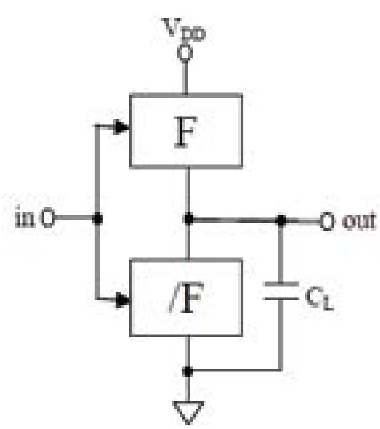

Fig.1: Conventional CMOS logic circuit with pull-up (F) and pull-down (/F) circuit [8].

III. ADIABATIC LOGIC

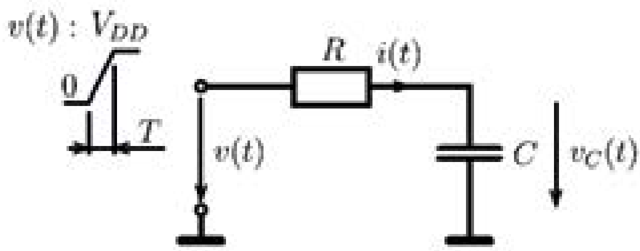

Fig. 2: Adiabatic Charging

To calculate the energy consumed by charging a capacitance adiabatically, the equivalent circuit is shown in Fig. 2. Here, the load capacitance *C* is charged by a constant current source *i(t)*. In conventional *CMOS* logic, constant voltage source is used to charge the load capacitance. Here, R is the on-resistance of *PMOS* network [9][10].

Therefore the current into the circuit can be determined by-

$$i(t) = \frac{Cdv(t)}{d(t)} = \frac{CV_{DD}}{T}$$

The energy for a charging event is calculated by integrating the power p(t) during the transition time T.

$$E = \int_0^T p(t)dt = \int_0^T v(t).i(t)\,dt$$

$$E = \int_0^T \left(V_R(t) + V_C(t)\right).i(t)\,dt$$

Since no energy is dissipated in the capacitor at one clock cycle. Therefore energy expression becomes-

$$E = \int_0^T RC^2 \frac{V_{DD}^{\ 2}}{T^2}$$

$$E = \frac{RC}{T} CV_{DD}^{\ 2}$$

During recovery process the same amount of energy is wasted. Therefore, the total energy dissipation over complete cycle is given as-

$$E = \frac{2RC}{T} CV_{DD}^{\ 2}$$

From the above expression the energy loss is inversely proportional to the switching time T. Here, the interesting thing is that the energy consumption is not only govern by the time period T but also the resistance R which is absence in the conventional CMOS. Thus if *2>>2RC* then, the energy dissipation is lesser than the conventional CMOS [8],[2].

IV. PROPOSED ADIABATIC INVERTER

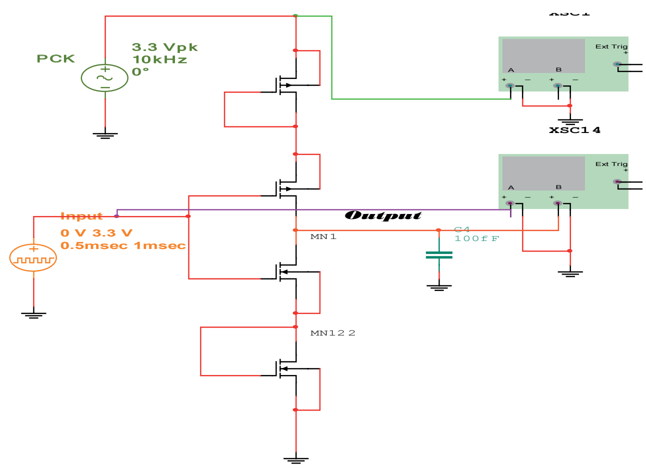

Fig. 3: Proposed Adiabatic Inverter

The basic inverter circuit is shown in the Fig. 3. It consists of a single phase power supply Power Clock *(PCK)* and four transistors in which two of them are *PMOS* and the other two are *NMOS*.

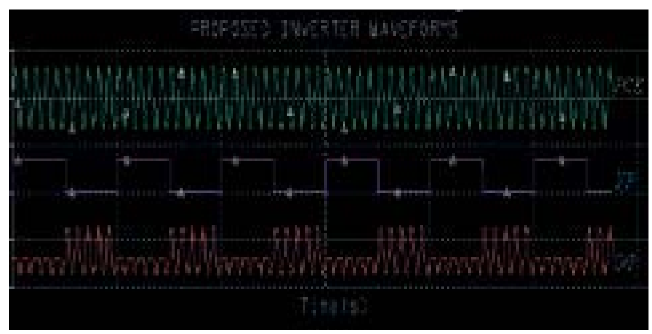

Fig.4: Waveforms of the Proposed Adiabatic Inverter

The circuit is being derived from traditional CMOS and is driven by a single power supply PCK. As seen from the above Fig.3, two more transistors are added to the conventional CMOS inverter in such a way that PMOS and NMOS are connected to pull-up and pull-down sides. The power clock frequency is set a little bit higher than usual so as to get better output logic. During evaluation phase, the power supply PCK swings up and is followed by the output according to the logics and in the recovery phase PCK swings down and the voltage stored at the output capacitor are sent it back to the supply PCK as shown in the Fig.4. Hence the energy got recovered from the output node.

V. CIRCUIT IMPLEMENTATION

Adders are the basic building blocks of all arithmetic circuits; adders add two binary numbers and give out sum and carry as output. Since adders are needed to perform arithmetic, they are an essential part of any computer. In order to cascade several one-bit adders to configure multiple bit adders there must be a provision to add the carry from the previous stage.

Such an adder is called a full adder. It is possible to create a logical circuit using multiple full adders to add N-bit numbers. Each full adder inputs a C_{in} which is the C_{out} of the previous adder. This kind of adder is called a ripple-carry adder, since each carry bit "ripples" to the next full adder. Four full adders can be cascaded to configure a 4-bit adder. The arrangement is shown in fig.9. Boolean expression, circuits and corresponding simulated waveforms are given below. Here Fig.5 & Fig.7 shows the full adder circuits for proposed and 2PASCL techniques. Fig.6 & Fig.8 shows the simulated waveforms of proposed & 2PASCL based designed full adder circuits. Proposed power efficient Four bit ripple carry adder has been configured by cascading 4-stages of proposed single-bit full adder circuit. The block diagram and it corresponding simulated waveforms are shown in Fig.9 & Fig.10.

$$Sum = \overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$$

$$Carry = AB + BC + AC$$

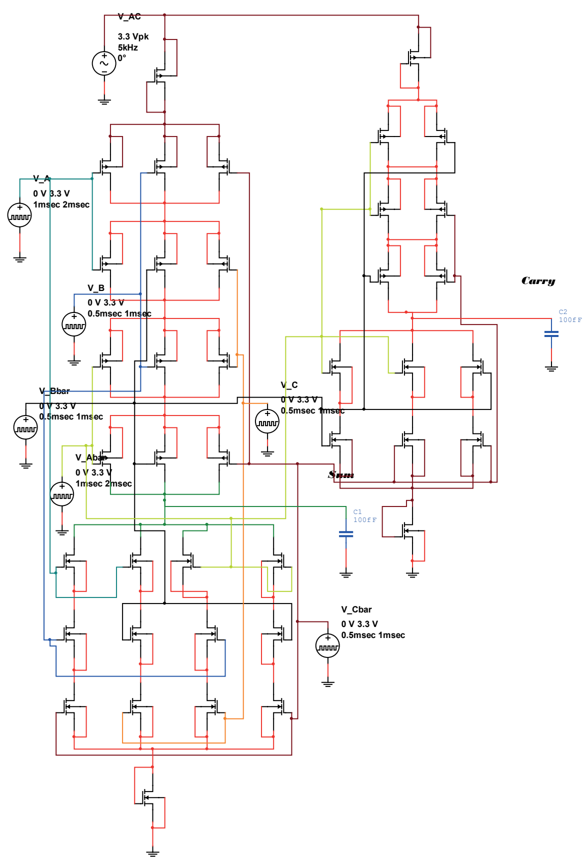

Fig.5 Proposed Full Adder Circuit

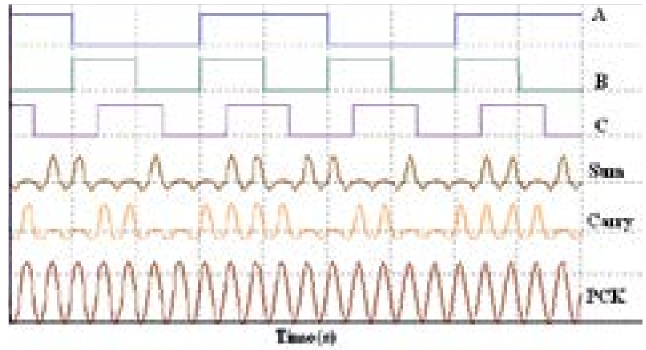

Fig.6 Simulated Proposed Full Adder Circuit Waveforms

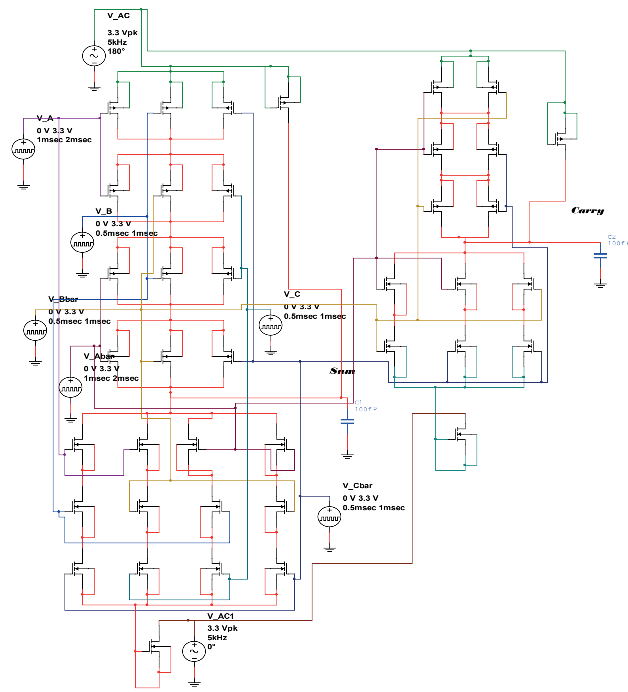

Fig.7 2PASCL Full Adder Circuit

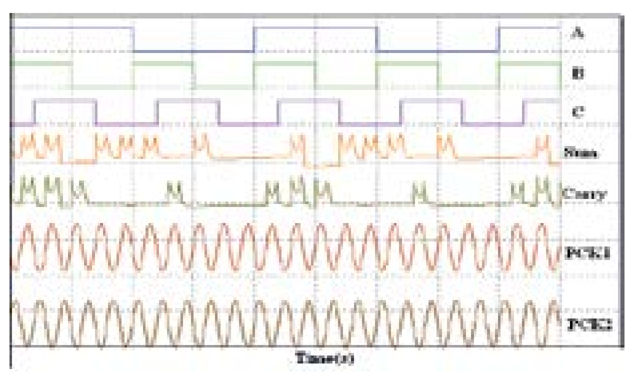

Fig.8 Simulated 2PASCL Full Adder Circuit Waveforms

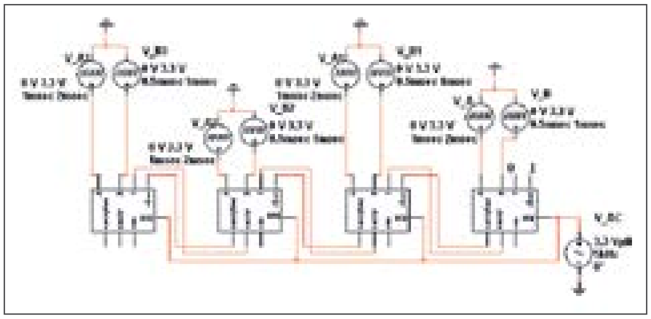

Fig.9 Configured proposed 4-Bit Ripple Carry Adder Block by Cascading 4-Stages of 1-Bit Full Adder

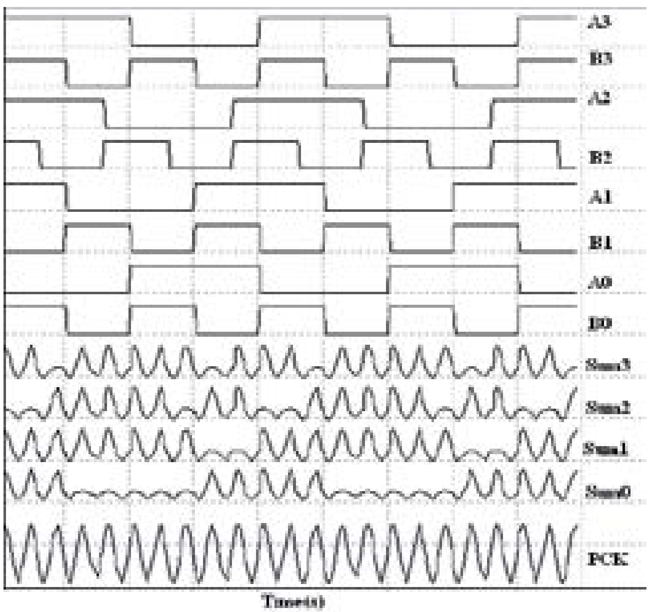

Fig.10 Simulated Proposed 4-Bit Ripple Carry Adder Circuit Waveforms

VI. POWER CONSUMPTION ANALYSIS & COMPARISON

Estimation of power consumptions is carried out at 0.5µm technology with the W/L ratio of the PMOS and NMOS are same and taken as L=0.5µm and W=1.25µm. The simulation has been done in NI Multisim with VDD=3.3V and load capacitance =100fF at a frequency range of 200 to 800MHz. 2PASCL and Proposed power efficient Adiabatic logic are investigated against the conventional CMOS logic for full adder circuits. Simulated power plots of the full adder circuits is shown in Fig .11

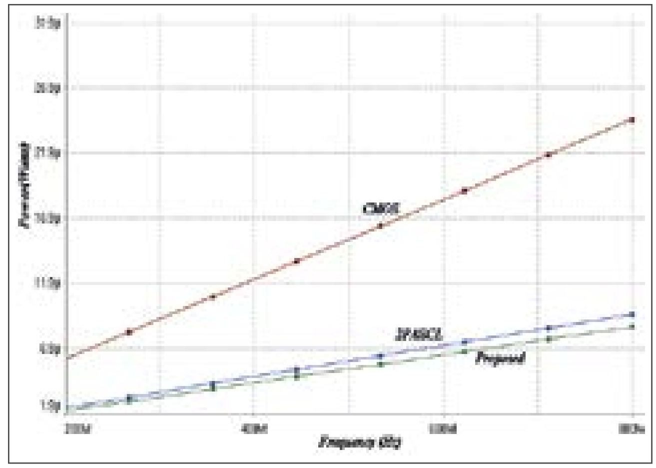

Fig.11 Simulated Power plot of Full Adder circuits

VII. CONCLUSION

In this paper, 4-bit ripple carry adder (RCA) circuit has been designed based on the proposed adiabatic logic technique & it corresponding simulated waveforms is being presented. The power consumption has also been calculated on a Full Adder circuits. The circuit diagrams and simulated output waveforms are presented and the power dissipations of the circuits are evaluated and compared with the counterpart conventional CMOS circuit. From the above observations, it can be concluded that the proposed power efficient adiabatic logic circuits give superior performance and shows 87.5% energy saving at 400MHz when compared to traditional methods. The proposed power efficient adiabatic logic circuit will be advantageous for ultra low energy computing applications.

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A REVIEW ON FRACTAL GEOMETRY FOR MULTIBAND ANTENNA APPLICATION

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ABSTRACT

Fractal geometries can be implemented to minimize the antenna size compared to the normal antenna, where the self similar nature of different fractal geometry can be applicable for multi band application. The designed antenna is used not only to get size reduction, but also to get changed frequency. There are many techniques to improve the characteristic of antennas. In this review we mainly discussed on different structure iteration for better antenna application.

Key words: - Fractal antenna, Multi band, miniaturized structure.

I. NTRODUCTION

The word fractal means an object, which is indefinitely divided. Its Latin name is "fractus" that descends from the verb "frangere", which means to break. It is now more than a decade in which geometrical characteristics of fractals are being applied in the design of passive components in the RF and microwave domain. Fractal antenna design paradigm is still in its infancy. Fractal geometries in small antennas are the order associated with these geometries in contrast to an arbitrary meandering of random line segments (which may also result in small antennas). Benoit B. Mandelbrot [1] showed that many fractals existed in nature and that fractals could accurately model certain phenomena. Simulation and implementation for experimental study in identifying features of fractal shaped antennas that could impart increased flexibility in the design of newer generation wireless systems. For reducing size of resonating antenna fractal geometry is the most effective geometry. Different types of curves geometry like Koch curves[2], Murkowski curves[3] , FASS curves [4] the Koch dipole, Koch monopole, Koch loop, and Minkowski loop has been described for getting longer features within small and compact area. First ever usage of the terminology "Fractal Antenna" in published document [5], this article also state that fractal geometry was first used to design frequency selective surfaces. Cohen had reported properties of fractal based antenna in different research papers [6-11]. Traditionally, a wideband antenna in the low frequency wireless bands can only be achieved with heavily loaded wire antennas, which usually means different antennas, are needed for different frequency bands. Recent progress in the study of fractal antennas suggests some attractive solutions for using a single small antenna operating in several frequency bands [12]. Cohen [11] was the first to develop an antenna element using the concept of fractals. He demonstrated that the concept of fractal could be used to significantly reduce the antenna size without de generating the performance. Fractals have self-similarity, so fractal antenna elements or arrays also can achieve multiple frequency bands due to the self-similarity between different parts of the antenna. Antenna based on fractal geometry has carved out a niche of itself with its inclusion as separate chapters in newer editions of popular antenna texts [13, 14]. In this review we had focused on different types of structure called fractal, which are applicable for antenna application.

"Antenna will provide vital links to and from everything out there. The future of antennas reaches to the stars "….John D Kraus

II. FRACTAL DIMENSIONS

Properties, bandwidth and radiation characteristics can be changed by changing the size or shape. In fractal geometry there is different iteration from where it is possible to get different radiation pattern. Iterated Function System (IFS) algorithm [17] generates a fractal curve also a set of transformation forms the IFS for the generation of the fractal structures. Now consider N (δ) boxes, of linear size δ, are necessary to cover a set of points distributed in a plane, then the box dimension is defined as the power D in the equation 1,

$$N(\delta)=\delta^{-D} \tag{1}$$

By taking log of both sides we can define the power for solving the iteration it can be written as

$$D = \frac{\mathrm{Log}\,[\mathrm{N}(\delta)]}{\mathrm{Log}\frac{1}{\delta}} \tag{2}$$

III. THE KOCH SNOWFLAKE

Now consider a straight line P_0P_1 , where length is one, now by considering 1st iteration the length will be $L=\frac{4}{3}$, all the line are equal size and shape (Fig.b) where p_a, p_b, p_c are the three iteration points. After taking 2nd iteration length will be $L=(4/3)^2$. After taking 3rd iteration the length will be $L=(4/3)^3$ so by changing only the power for taking the different no of iteration as a result for ∞ no of iteration length will be $L=(4/3)^{\infty}=\infty$ (Shown in figure e).

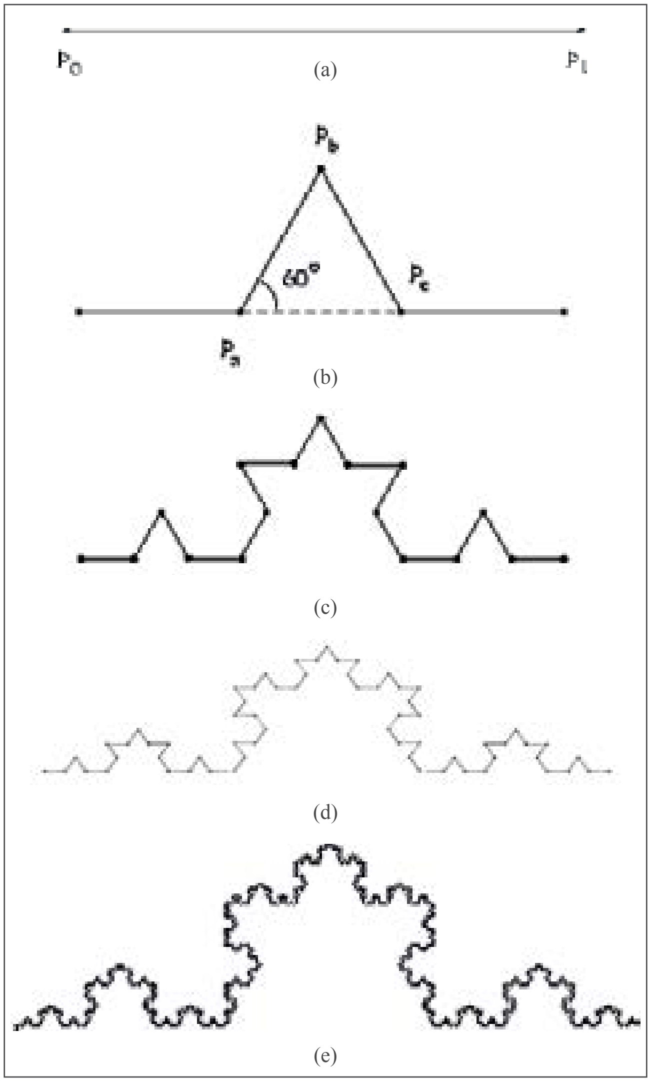

Figure 1: Different iteration (a) original, (b),(c),(d),(e) are 1st , 2nd, 3rd , and nth iteration.

Here discussed pattern is called The Koch snowflake, by taking six of these structures and put together to form the structure like infinite (figure f) but that the area it bounds is finite (indeed, it is contained in the white square) where each of the six sides of the Koch snowflake is self-similar if you take a small copy of it.

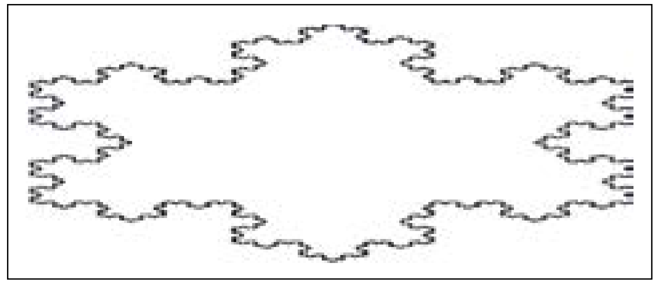

Figure f: Infinite iteration

But self-similarity is not what makes the Koch snowflake a fractal! (Contrary to a common misconception) After all, many common geometric objects exhibit self-similarity, consider, for example, the humble square.

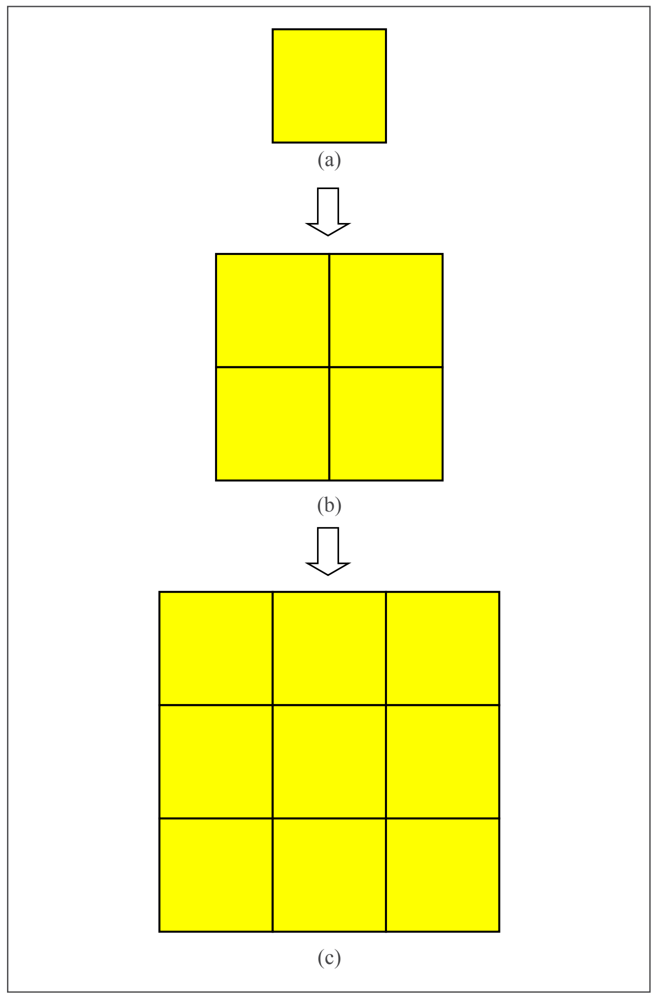

Figure: 2 Different iteration by taking square (a),(b),(c) are original square, dilate by factor 2, dilate by factor 3.

Now consider a small square and dilate by a factor of 2 then it is possible to get 4 copies of the original. A square is self-similar, but it most certainly is not a fractal. Also it is possible to define different structure by using scale factor.

Consider scale factor is K, Let N be the number of copies of the original that is possible to get. By considering squire if can be define like $K^2=N$ or it can be expressed by using logarithmic term as $Log_K N=2$. So as a whole for all the structure it can be written as $Log_K N$. Now consider another shape triangle and taking scale factor (k) is 2, so no of copies of original (N) will be 4 and for that case logarithmic term will be $Log_K N=2$

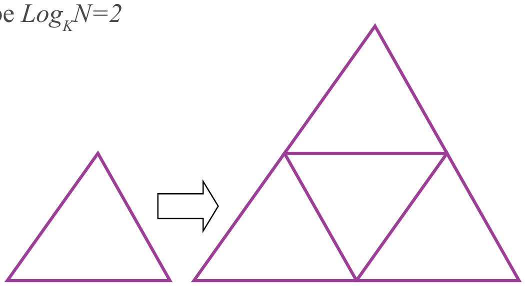

Figure 3: a (original triangle), b (scale factor 2)

Same iteration can also be considered by taking structure cube also (shown in figure 4). Where by considering scale factor 2 and number of copies of original (N) 8, the value of $Log_K N$ will be 3. $Log_K N$ tells that the dimensions of shape where the shape has to be self-similar. It turns out that this definition coincides with a much more general definition of dimension

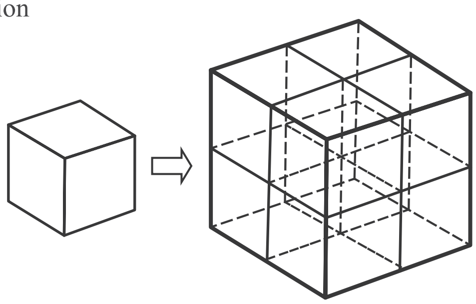

Figure 4: Iteration of cube by scale

called the fractal dimension. Now let's form Koch snowflake structure where by considering scale factor (k) is 3 and number of copies of original (N) is 4 so the will be which is 1.261. So each side of the Koch snowflake is approximately 1.261-dimensional that's what makes the Koch snowflake a fractal – the fact that its dimension is not an integer. Even shapes which are not self-similar can be fractals. The most famous of these is the Mandelbrot set.

Shape	$Log_K N$
Square	2
Line segment	1
Triangle	2
Cube	3

Table 1: Changing of $Log_K N$ value by considering different shape

IV. THE SIERPINSKI CARPET

In this technique of calculation we define the antenna structure from taking square configuration. Start with a square of side length 3, with a square of side length 1 removed from its center (Shown in figure). So the perimeter (P) will be [4(3)+4(1)] and the area (A) is [32-12].

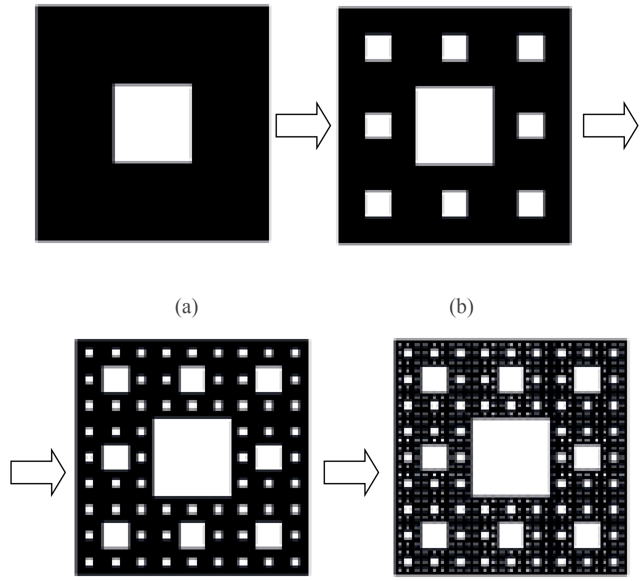

Figure 5: Different iteration from 1 to n

Again when we will consider this shape as consisting of eight small squares, each of side length 1 and from each small square, remove its central square. Then perimeters $(p)=[4\cdot 3+4\cdot 1+8\cdot 4\cdot \frac{1}{3}]$ and area $(A)=[3^2-1^2-8\cdot\left(\frac{1}{3}\right)^2]$ So from n no of iteration the perimeter (P) will be

$$P=4\cdot 3+4\cdot 1+8\cdot 4\cdot \frac{1}{3}+8^2\cdot 4\cdot\left(\frac{1}{3}\right)^2+8^3\cdot 4\cdot\left(\frac{1}{3}\right)^3+\ldots$$

$$=4\cdot 3+\sum_{n=0}^{\infty}4\cdot\left(\frac{8}{3}\right)^n=\infty$$

And the area (A) will be

$$A=3^2-1^2-8\cdot\left(\frac{1}{3}\right)^2-8^2\cdot\left(\frac{1}{3}\right)^{2\cdot 2}-8^3\cdot\left(\frac{1}{3}\right)^{2\cdot 3}-\ldots$$

$$=3^2-\left[1+\frac{8}{9}+\left(\frac{8}{9}\right)^2+\left(\frac{8}{9}\right)^3+\ldots\right]=3^2-\left(\frac{1}{1-\frac{8}{9}}\right)=0$$

So the Sierpinski carpet has an infinite perimeter-but it bounds a region with an area of zero and fractal dimension of the Sierpinski carpet will be . The Sierpinski carpet has a 3-dimensional analogue called the Menger sponge. Its surface area is infinite, yet it bounds a region of zero volume. The fractal dimension of the Menger sponge is . Also it may be iterate by using different structure and different established methods.

V. SCOPE OF WORK

A novel CPW feed complementary Sierpinski carpet planar monopole antenna can be investigated and analysed that has been performed using Real Coded Genetic Algorithm (RCGA) which can be accomplished using Particle Swarm Optimization (PSO). Only changing the shape of ground plane rather than perturbing the geometry itself Sierpinski carpet planar monopole antenna with CPW feed can be optimized and characterized for different better result. In fractal wire antenna domain the RCGA based optimization can be applied by linking Numerical Electromagnetic Code (NEC) with MATLAB™ this is well described by Derek Linden [15]. Also some studies on Sierpinski fractal loop antenna has been undertaken and reported in [16] on which we can do different studies.

VI. CONCLUSION

Wideband and low profile antennas are in great demand for both commercial and military applications. The miniaturized structure of fractal antenna can be fabricated inside a small device and its multi-band and wideband antennas application are desirable in personal communication systems, small satellite communication terminals, and other wireless applications. Most fractals have infinite complexity and detail makes it possible to use fractal structure to design small size, low profile, and low weight antennas.

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INVESTIGATION OF NEW SCHEME OF AGGRESSIVE PACKET COMBINING

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ABSTRACT

Aggressive Packet Combining (APC) scheme is well established in literature for receiving correct packet in high error prone wireless link. In APC three copies of a packet are transmitted and receiver does bit wise majority decision to get correct copy. Major research challenge of the APC is that in APC the minimum average number of times a packet needs to be transferred from a source to a destination for successful reception of the packet is 3, we propose a new protocol of transmission where average number of transmission can be reduced and new protocol studied /reported mathematically and experimentally.

Keywords : Packet Combining Scheme, Conventional Aggressive Packet Combining Scheme (CAPC), Throughput, Bit error rate, sequential mode.

I. NTRODUCTION

In order to combat errors in computer/ data communication networks, ARQ (Automatic Repeat Request) techniques [1-5] with various modifications as applicable to in various communication environments are used. Leung [7] proposed an idea of Aggressive Packet Combining scheme (APC) for error control in wireless networks with the basic objective of fast error control in relatively higher noisy wireless networks. APC is well established and studied elsewhere [3-10].Several modifications of APC are also reported elsewhere [2-13]. The modifications are due to increasing throughput, tackling various error syndromes and enhancing fast correction. In APC and/ or modified APCs, two or more copies of the packets are transmitted. Copies received by the receiver either error free or erroneous are used in receiver to correct errors by applying Packet Combining schemes differently in different situations. However in original APC, if an error at same locations of erroneous packets, the application of the majority logic as an original APC fails to correct the error. To address the stated problem of APC we propose two new protocols of APC. Analytical results establish that the proposed new schemes are superior to original APC.

II. REVIEW OF PACKET COMBING SCHEME

Chakrabotry [11] suggested a very simple and elegant technique known as packet combining scheme (PC) where the receiver will correct limited error, one or two bit error, from the received erroneous copies. As per Chakraborty's proposal:

It is assumed that an original packet "10101010" is to be transmitted between a sender and a receiver. The packet erroneously received by the receiver is "00101010". The receiver requests for retransmission of the packet and keeps the copy that has been received erroneously as well. The transmitter retransmits the packet, but again the packet is received by the receiver erroneously as "11101010". Chakraborty suggested that the receiver can correct the error by using two erroneous copies. After making a bit wise XOR operation between erroneous copies, the receiver can locate the error position. The operation can be identified by an example given below:

First erroneous copy	=	00101010
Second erroneous copy	=	11101010
XOR		11000000

The error locations are identified as first and/or second bit from left. Chakraborty suggested that the receiver can apply the brute method to correct error by changing received "1" to "0" or vice versa on the received copies followed by the application of error decoding method in use. In the example the average number of brute application will be ½ and in general 2n-1 if n bits are found in error. Several modifications of PC have been studied elsewhere [12-13] by Bhunia's.

III. REVIEW OF CONVENTIONAL APC

Aggressive packet combining scheme is a modification of MjPc (Majority Packet Combining) [14].To illustrate APC it is assumed that an original packet 10101010 is transmitted between a sender and a receiver. In Aggressive Packet combining Scheme (APC) the three copies of packet are sent for each packet between a source and a destination. The majority logic is applied bit to bit on three copies of packet. In table: (1) we have shown different possibilities of APC. In Case (1) there is no error in transmitted three copies. In Case (2) receiver receives two copies of correct packet and one copy with an error, so the correction is possible by majority logic. In Case (3) and Case (4) errors are present in two or more copies in which case correction is not possible.

Table: 1: Different cases of Aggressive Packet Combining Scheme

Case 1	Case 2	Case 3	Case 4
Copy-1= 10101010 Copy -2= 10101010 Copy-3= 10101010	Copy-1= 00101010 Copy -2= 10101010 Copy-3= 10101010	Copy-1= 00101010 Copy -2= 00101010 Copy-3= 10101010	Copy-1= 00101010 Copy -2= 00101010 Copy-3= 00101010
Correction Probability is (1-P3)	Correction Probability is (1-P2) P	Correction Probability is (1-P) P2	Correction Probability is (P3)
Correction not required.	Correction possible	Correction not possible	Correction not possible

IV. NEW BASIC IDEA

In APC the minimum average number of times a packet needs to be transferred from a source to a destination for successful reception of the packet is 3. To increase the throughput, we like to propose a variation to the limitation of APC. A protocol is illustrated below up to two bits error:

Step I: The sender will transmit two copies of the packet to the receiver first. Then receiver will do bit wise XOR with these two copies. As discussed in Packet Combining Scheme and if error locations are identified (up to two bits) then perform Step II otherwise accepts these copies as correct copy and send positive acknowledgement to the sender.

Step II: Through a secure feedback path receiver will transmit identified erroneous bit locations to the sender.

Step III: Sender likes to transmit correct bit value from original copy up to four times for each single bit error.

Step IV: Lastly receiver will generate original bit from transmitted bits to generate correct copy.

The scheme is illustrated properly by taking different examples; suppose original copy is “10101010” and receiver receives two erroneous copies as “00101010” and “10101110” (error places marked as boldfaces).

1st copy: 00101010
2nd copy: 10101110

XOR 10000100

From above XOR operation we can easily identify two erroneous bits are 1st and 6th from left. Now receiver will transmit this erroneous bit’s information to the sender through a secure path then sender will send “1111” and “0000” respectively for 1st and 6th bit locations, by these values receiver can easily generate original copy.

Suppose another original copy is “11000101”and receiver receives two copies as “11000111” and “11000101”.

1st copy: 11000111
2nd copy: 11000101

XOR 0000010

So in this case erroneous bit is in 7th bit from left. After getting this information from the receiver, sender sends “0000” to the receiver for the generation of correct copy.

V. ANALYSIS

Average number of packet transmission in conventional APC to get correct copy is 3 but in proposed scheme average number of packet transmission is;

M = 2 + [nC2 . α2 (1-α)n-2] * 2 , where α is the bit error rate and n is total number of bits. Fig (1) is drawn with different values of n (e.g: 8,16,32,48) in respect of different BER(10-1 to 10-3). And can be strongly inferred that proposed scheme is quite superior to APC.

Also we have conducted an experiment with 8 bit data (0000000 to 11111111) with different one bit and two bit error vectors and we have got 100% error correcting capability in proposed scheme.

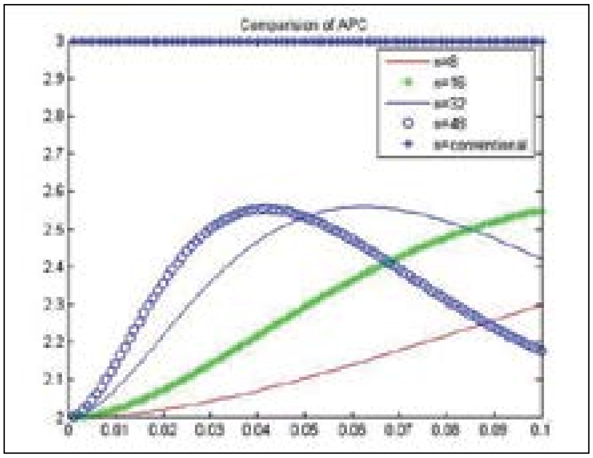

Fig: (1) Comparison of APC with proposed scheme.

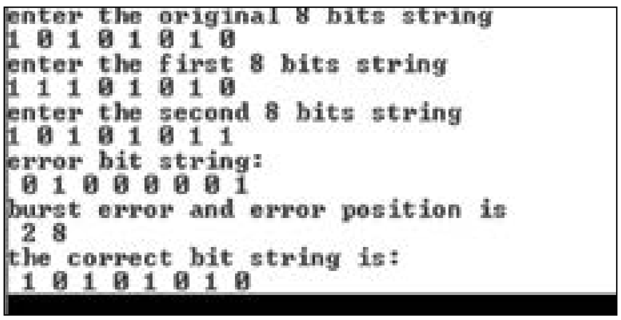

Fig: (2) Partial simulation result.

VI. CONCLUSIONS

In this paper, a new scheme of APC are proposed & studied. These schemes provide better correction capability. Simulation study will be made in future research.

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APPROXIMATE ANALYTICAL SOLUTIONS TO FRACTIONAL INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

In this paper, we present an efficient numerical method to solve fractional integro-differential equation based on the differential transform method of Odibat et al. [1]. The proposed method is applied to solve Caputo fractional integro-differential equation combined with initial conditions. Numerical examples are investigated to illustrate the effectiveness of the generalization.

Keywords - erivatives and integrals; fractional differential equation; differential transform method.

I. NTRODUCTION

In the past decade, both mathematicians and physicists have devoted considerable efforts to study various schemes for the solution of linear and nonlinear differential equations of fractional order. These schemes can be broadly classified into two classes viz. analytical and numerical. Many models considering the fractional differential operators are described in [6, 9, 13, 14]. More recent applications of linear and nonlinear multi-order fractional differential equations can be found in [1]. The numerical solution of fractional differential equations has been a standard topic for the applied problem. According to the authors [2, 3, 7], the numerical schemes can be further divided into two groups: the solution is approximated over the entire domain using approximating functions such as polynomials and orthogonal functions; and the entire domain is divided into several small domains like in a finite element technique, and the solution is obtained for variables.

In a series of papers [4, 11] the authors have been solved linear and nonlinear differential equations of fractional order. This method is based on the differential transform method and generalized Taylor's formula [11]. Further, Hwang et al. [12] generalized the differential transform method and applied the method to study the problem arises in the nonlinear optimal control. In the present paper, we extend a semi-numerical method based on the one-dimensional generalization of differential transform method to the integro-differential equation of fractional order. In many applications, the differential equations of fractional order in Caputo's sense provide more accurate models of system under consideration.

II. REVIEW ON FRACTIONAL OPERATORS

Fractional calculus is a generalization of integration and differentiation of fundamental operator to a fractional, or non-integer order. We first introduce some definitions and properties of the fractional calculus [5].

Definition 1.1. Let $f(t)$ be a real function defined on Ω, where $\Omega = [a,b]$ be a finite interval on the real axis R.

The left-sided Riemann-Liouville fractional integral $\mathcal{I}_{a+}^{\alpha}$ of order $\alpha \in C(R(\alpha) > 0)$ is defined by

$$\left(\mathcal{I}_{a+}^{\alpha} f\right)(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau. \qquad (1.1)$$

The semi group property satisfies by the operator as follows

$$\mathcal{I}_{a+}^{\alpha} \mathcal{I}_{a+}^{\beta} = \mathcal{I}_{a+}^{\alpha+\beta} = \mathcal{I}_{a+}^{\beta} \mathcal{I}_{a+}^{\alpha}$$

for all $\alpha, \beta \geq 0$.

Definition 1.2. For $f(t) \in \mathbf{C}_{\alpha}^{m}$, $m \in N$, the left-sided Caputo's fractional derivative of $f(t)$ is defined by

$$ {}^{C}\mathcal{D}_{a+}^{\alpha} f(t)=\begin{cases}\mathcal{I}_{a+}^{m-\alpha} f^{(m)}(t), & m-1<\alpha<m,\\ D^{m} f(t), & \alpha=m,\\ \mathcal{I}_{a+}^{\beta} f(t), & \alpha=-\beta\leqslant 0.\end{cases} \quad (1.2) $$

Therefore, for $\alpha>0$ and $\beta>m$

$$ {}^{C}\mathcal{D}_{a+}^{\alpha}(t-a)^{\beta}=\frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(t-a)^{\beta-\alpha}, \quad (1.3) $$

and

$$ {}^{C}\mathcal{D}_{a+}^{\alpha}(t-a)^{k}=0, \qquad (k=0,1,\cdots,m-1). $$

Proposition 1.3. Let $m-1<\alpha\leqslant m$, $f(t)\in\mathbf{C}_{\alpha}^{m}[a,b]$, and $\alpha\geqslant -1$ we have

$$ \left(\mathcal{I}_{a+}^{\alpha}\, {}^{C}\mathcal{D}_{a+}^{\alpha} f\right)(t)=f(t)-\sum_{k=0}^{m-1}\frac{f^{(k)}(a)}{k!}(t-a)^{k} \quad (1.4) $$

The following result is very useful for solving differential equation of fractional order [10, 1].

Theorem 1.4. Suppose that $f(t)=t^{\lambda}g(t)$, where $\lambda>-1$ and $g(t)$ has generalized power series expansion $g(t)=\sum_{k=0}^{\infty}a_k(t-a)^{k\alpha}$ with radius of convergence $\mathbf{R}>0$, $\alpha\in(0,1]$. Then

$$ \left({}^{C}\mathcal{D}_{a+}^{\gamma}\, {}^{C}\mathcal{D}_{a+}^{\beta} f\right)(t)=\left({}^{C}\mathcal{D}_{a+}^{\gamma+\beta} f\right)(t), \quad (1.5) $$

for all $t\in(0,\mathbf{R})$ if any of the following condition is to be satisfies:

i. $\beta<\lambda+1$ and α arbitrary,

ii. $\beta\geqslant\lambda+1$ and γ arbitrary, and $a_k=0$ for $k=0,1,\cdots,n-1$, where $n-1<\beta\leqslant n$.

We will note that in general the Caputo's fractional derivative introduced in (1.2) is not commutative.

Definition 1.5. Let $\alpha\in\mathbb{R}^{+}$, $\Omega\subset\mathbb{R}$ an interval such that $a\in\Omega$, $a\leqslant t$ $\forall t\in\Omega$. Then the following sets of functions are defined:

$$ \mathbf{I}_{a}^{\alpha}=\left\{f\in\mathbf{C}_{\alpha}(\Omega): \mathcal{I}_{a+}^{\alpha} f(t) \text{ exist and is finite in } \Omega\right\}, $$

$$ \mathbf{D}_{a}^{\alpha}=\left\{f\in\mathbf{C}_{\alpha}(\Omega): {}^{C}\mathcal{D}_{a+}^{\alpha} f(t) \text{ exist and is finite in } \Omega\right\}. $$

The generalized Taylor's formula that involves the Caputo's fractional derivative was presented in [11, 13]. Before introducing the generalized Taylor's formula we begin with the generalized mean value theorem of Odibat et al. [11].

Theorem 1.6. (Generalized Mean Value Theorem) Suppose that $f(x)\in\mathbf{C}_{\alpha}[a,b]$ and ${}^{C}\mathcal{D}_{a+}^{\alpha} f(t)\in\mathbf{C}_{\alpha}(a,b]$, for $0<\alpha\leqslant 1$, then we have

$$ f(t)=f(a)+\frac{1}{\Gamma(\alpha)}\left({}^{C}\mathcal{D}_{a+}^{\alpha} f\right)(\xi)(t-a)^{\alpha} \quad (1.6) $$

With $a\leqslant\xi\leqslant t$, $\forall t\in(a,b]$.

When $\alpha=1$, the generalized mean value theorem reduces to the classical mean value theorem. We need the following proposition to present the generalized Taylor's formula in the Caputo's sense.

Proposition 1.7. Let $\alpha\in(0,1]$, $m,n\in\mathbb{N}$ and f(t) an analytic function on $\Omega\subset\mathbb{R}$, $f(t)\in\mathbf{D}_{a}^{(m+1)\alpha}$ with $a,t\in\Omega$, $a<t$, then we have

$$ \left(\mathcal{I}_{a+}^{m\alpha}\, {}^{C}\mathcal{D}_{a+}^{m\alpha} f\right)(t)-\left(\mathcal{I}_{a+}^{(m+1)\alpha}\, {}^{C}\mathcal{D}_{a+}^{(m+1)\alpha} f\right)(t) $$

$$ =\frac{(t-a)^{m\alpha}}{\Gamma(m\alpha+1)}\, {}^{C}\mathcal{D}_{a+}^{m\alpha} f(a) \quad (1.7) $$

Where ${}^{C}\mathcal{D}_{a+}^{m\alpha}=\underbrace{{}^{C}\mathcal{D}_{a+}^{\alpha}\cdot{}^{C}\mathcal{D}_{a+}^{\alpha}\cdots{}^{C}\mathcal{D}_{a+}^{\alpha}}_{m-\text{times}}$.

The above proposition would be the initial point to construct the power series of a sufficiently well behaved function $f(t)$.

Proposition 1.8. Suppose that the conditions of Proposition 1.7 are to be satisfied and ${}^{C}\mathcal{D}_{a+}^{m\alpha} f(t)\in\mathbf{C}_{\alpha}(a,t]$ and ${}^{C}\mathcal{D}_{a+}^{\alpha} f\in\mathbf{I}_{a}^{k\alpha}$, $m,k\in\mathbb{N}$, then

$$ \left(\mathcal{I}_{a+}^{k\alpha}\, {}^{C}\mathcal{D}_{a+}^{m\alpha} f\right)(t)=\frac{(t-a)^{k\alpha}}{\Gamma(k\alpha+1)}\left({}^{C}\mathcal{D}_{a+}^{m\alpha} f\right)(\xi) \quad (1.8) $$

for all $\xi\in(a,t)$.

Theorem 1.9. (Generalized Taylor's formula [1]) Suppose that ${}^{C}\mathcal{D}_{a+}^{k\alpha} f\in\mathbf{C}_{\alpha}(a,b]$ for $k=0,1,\cdots,m+1$, where $0<\alpha\leqslant 1$, then we have

$$ f(t)=\sum_{i=0}^{m}\frac{(t-a)^{i\alpha}}{\Gamma(i\alpha+1)}\left({}^{C}\mathcal{D}_{a+}^{i\alpha} f\right)(a)+\frac{\left({}^{C}\mathcal{D}_{a+}^{(m+1)\alpha} f\right)(\xi)}{\Gamma((m+1)\alpha+1)}(t-a)^{(m+1)\alpha}, \quad (1.9) $$

with $a\leqslant\xi\leqslant t$ for all $t\in(a,b]$, where ${}^{C}\mathcal{D}_{a+}^{m\alpha}=\underbrace{{}^{C}\mathcal{D}_{a+}^{\alpha}\cdot{}^{C}\mathcal{D}_{a+}^{\alpha}\cdots{}^{C}\mathcal{D}_{a+}^{\alpha}}_{m-\text{times}}$.

In the case of $\alpha=1$, the generalized Taylor's formula in Caputo's sense (1.9) reduces to the classical Taylor's formula. The radius of convergence $\mathbf{R}$ of the generalized Taylor's series for the function

$$ f(t)=\sum_{k=0}^{\infty}\frac{(t-a)^{k\alpha}}{\Gamma(k\alpha+1)}\left({}^{C}\mathcal{D}_{a+}^{k\alpha} f\right)(\xi), \quad (1.10) $$

Depends on $f(t)$ and a, and is given by $\mathbf{R} = \left|(t-a)^{\alpha}\right| \lim_{m\to\infty} \left| \frac{\Gamma(m\alpha+1)}{\Gamma((m+1)\alpha+1)} \cdot \frac{({}^{C}\mathcal{D}_{a+}^{(m+1)\alpha} f)(a)}{({}^{C}\mathcal{D}_{a+}^{m\alpha} f)(a)} \right|$.

(1.11)

Theorem 1.10. Suppose that ${}^{C}\mathcal{D}_{a+}^{k\alpha} f \in \mathbf{C}_{\alpha}(a,b]$ for $k = 0,1,\cdots,m+1$, where $0 < \alpha \leqslant 1$, then

$$f(t) \cong P_N^{\alpha}(t) = \sum_{i=0}^{N} \frac{(t-a)^{i\alpha}}{\Gamma(i\alpha+1)} ({}^{C}\mathcal{D}_{a+}^{i\alpha} f)(a), \tag{1.12}$$

and there is a value ξ with $a \leqslant \xi \leqslant t$ so that the error term R_N^{α} has the form:

$$R_N^{\alpha}(t) = \frac{({}^{C}\mathcal{D}_{a+}^{(N+1)\alpha} f)(a)}{\Gamma((N+1)\alpha+1)} (t-a)^{(N+1)\alpha} \tag{1.13}$$

Hence, the accuracy of the $P_N^{\alpha}(t)$ increases when we choose large N and decreases as the value of t moves away from the center a. Thus, we must choose N large enough so that the error does not exceed a specified bound. For more details see [11].

III. GENERALIZED DIFFERENTIAL TRANSFORM METHOD

The differential transformation, like the well-known integral transformations (Fourier and Laplace transformation), is a linear operator that transforms a function from the original time and/or space domain into a transformed domain in order to minimize the differential calculations. But the differential transformation is different from the integral transformations in that the images of a function are determined by differential operations instead of integral operations [12].

In this section, we consider the fractional nonlinear integro-differential equation

$$y^{(\alpha)}(t) = f(t, y(t), y^{(\alpha_1)}(t), y^{(\alpha_2)}(t), \cdots, y^{(\alpha_k)}(t),$$
$$y^{(-\alpha_1)}(t), y^{(-\alpha_2)}(t), \cdots, y^{(-\alpha_k)}(t)), \tag{2.1}$$

Together with the initial condition

$$y(0) = y_0 \in \mathbb{R}, \tag{2.2}$$

Where

$y^{(\alpha_j)}(t) := ({}^{C}\mathcal{D}_{a+}^{\alpha_j} y)(t)$,

$y^{(-\alpha_j)}(t) := (\mathcal{I}_{a+}^{\alpha_j} y)(t), j = 1,2,\cdots,k$ and

$0 < \alpha - \alpha_j < \alpha + \alpha_j$, $j = 1,2,\cdots,k$; $m-1 < \alpha \leqslant m$, $m \in \mathbb{N}$.

The existence and uniqueness conditions for the nonlinear fractional integro-differential equation (2.1) and (2.2) were found in the recent paper of Matter [14].

The Taylor's series method is computationally takes long time for large order and requires symbolic computation of the necessary derivatives of the data function. The generalized Taylor's series method was well addressed in [11, 13].

The differential transform method was first introduced by Zhou [15], who solved linear and nonlinear initial value problems arising in electric circuit analysis. Generally, it was an iterative process to obtain analytical Taylor's series solution in the forms of a polynomial, which was different from the traditional higher order Taylor's series method. First we define the generalized differential transform of the k-th derivative of function $f(t)$ as follows:

$$F_{\alpha}(k) = \frac{1}{\Gamma(k\alpha+1)} \left[y^{(k\alpha)} f(t) \right] \tag{2.3}$$

where $y^{(k\alpha)}$ denotes the sequential fractional Caputo's derivative and the inverse differential transform of $F_{\alpha}(k)$ is defined as follows:

$$f(t) = \sum_{k=0}^{\infty} F_{\alpha}(k)(t-t_0)^{k\alpha} \tag{2.4}$$

It can easily show that (2.4) is the inverse of the generalized differential transform (2.3). In real applications, by using Theorem 1.10 we will approximate the function $f(t)$ by the finite series

$$f(t) = \sum_{k=0}^{n} F_{\alpha}(k)(t-t_0)^{k\alpha} \tag{2.5}$$

For the case $\alpha = 1$, the generalized differential transform (2.3) reduces to the classical differential transform of Zhou [15]. Equation (2.5) implies that the $\sum_{k=n+1}^{\infty} F_{\alpha}(k)(t-t_0)^{k\alpha}$ is negligibly small. In fact, n is decided by the convergence of natural frequency in this study. Some useful properties of generalized differential transform are introduced below:

Theorem 2.1. If $f(t) = g(t) \pm h(t)$, then $F_{\alpha}(k) = G_{\alpha}(k) \pm H_{\alpha}(k)$.

Theorem 2.2. If $f(t) = cg(t)$, then $F_{\alpha}(k) = cG_{\alpha}(k)$, where c is a constant.

Theorem 2.3. If $f(t) = g(t)h(t)$, then

$$F_{\alpha}(k) = \sum_{j=0}^{k} G_{\alpha}(j) H_{\alpha}(k-j).$$

Theorem 2.4. If $f(t)=\prod_{j=1}^{n} g_j(t)$, $j=1,2,\cdots,n,$ then

$$F_\alpha(k)=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\cdots\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2} G_{\alpha 1}(k_1)G_{\alpha 2}(k_2-k_1)\cdots G_{\alpha(n-1)}(k_{n-1}-k_{n-2})G_{\alpha n}(k-k_{n-1}). \quad 2.6)$$

Theorem 2.5. If $f(t)=g^{(\alpha)}(t)$, then

$$F_\alpha(k)=\frac{\Gamma((k+1)\alpha+1)}{\Gamma(k\alpha+1)}G_\alpha(k+1),$$

where $g^{(\alpha)}(t)$ is the Caputo's fractional derivative of $g(t)$.

Theorem 2.6. If $f(t)=(t-t_0)^\gamma$, $\gamma=n\alpha$, $n\in\mathbb{Z}$, then

$$F_\alpha(k)=\delta(k-\gamma/\alpha),$$

where $\delta(k)=\begin{cases}1 & \text{if } k=0,\\ 0 & \text{if } k\neq 0.\end{cases}$

Theorem 2.7. If $f(t)=g^{(\beta)}(t)$, $m-1<\beta\leqslant m$ and the function $g(t)$ satisfies the conditions of Theorem 1.4, then

$$F_\alpha(k)=\frac{\Gamma(k\alpha+\beta+1)}{\Gamma(k\alpha+1)}G_\alpha(k+\beta/\alpha).$$

Theorem 2.8. If $f(t)=\int_{t_0}^{t} g(t)dt$, then

$$F_\alpha(k)=\frac{1}{k\alpha}G_\alpha(k-1/\alpha),$$

where $k\geqslant 1/\alpha$.

Theorem 2.9. If $f(t)=g(t)\int_{t_0}^{t} h(t)dt$, then

$$F_\alpha(k)=\sum_{k_1=1/\alpha}^{k}\frac{1}{k_1\alpha}H_\alpha(k_1-1/\alpha)G_\alpha(k-k_1),$$

where $k\geqslant 1/\alpha$.

Theorem 2.10. If $f(t)=\int_{t_0}^{t} h_1(t)h_2(t)\cdots h_n(t)dt$, then

$$F_\alpha(k)=\frac{1}{k\alpha}\sum_{k_{n-1}=0}^{k-1/\alpha}\sum_{k_{n-2}=0}^{k_{n-1}}\cdots\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2} H_{\alpha 1}(k_1)H_{\alpha 2}(k_2-k_1)\cdots H_{\alpha(n-1)}(k_{n-1}-k_{n-2})H_{\alpha n}(k-k_{n-1}-1/\alpha),$$

where $k\geqslant 1/\alpha$.

Theorem 2.11. If $f(t)=g_1(t)g_2(t)\cdots g_m(t)\int_{t_0}^{t} h_1(t) h_2(t)\cdots h_n(t)dt$, then

$$F_\alpha(k)=\sum_{k_1=1/\alpha}^{k}\frac{1}{k_1\alpha}\sum_{j_{n-1}=0}^{k_1-1/\alpha}\sum_{j_{n-2}=0}^{j_{n-1}}\cdots\sum_{j_2=0}^{j_3}\sum_{j_1=0}^{j_2}\sum_{i_{m-1}=0}^{k-k_1}\sum_{i_{m-2}=0}^{i_{m-1}}\cdots\sum_{i_2=0}^{i_3}\sum_{i_1=0}^{i_2} G_{\alpha 1}(i_1)G_{\alpha 2}(i_2-i_1)\cdots G_{\alpha(m-1)}(i_{m-1}-i_{m-2})$$
$$\times G_{\alpha m}(k-i_{m-1}-k_1)H_{\alpha 1}(j_1)H_{\alpha 2}(j_2-j_1)\cdots$$
$$\times H_{\alpha(n-1)}(j_{n-1}-j_{n-2})H_{\alpha n}(k-j_{n-1}-1/\alpha),$$

where $k\geqslant 1/\alpha$.

Theorem 2.12. If $f(t)=\prod_{j=1}^{n} g_j^{(\alpha_j)}(t)$, $m_i-1<\alpha_i\leqslant m_i$, $m_i\in\mathbb{N}$, $i=1,2,\cdots,n$ and the function $g_j(t)$ satisfies the conditions of Theorem 1.4, then

$$F_\alpha(k)=\prod_{j=2}^{n}\sum_{k_{j-1}=0}^{k_j}\frac{\Gamma((k_j-k_{j-1})\alpha+\alpha_j+1)}{\Gamma((k_j-k_{j-1})\alpha+1)}$$
$$.\times G_{\alpha j}\left(k_j-k_{j-1}+\frac{\alpha_j}{\alpha}\right)G_{\alpha 1}\left(k_1+\frac{\alpha_1}{\alpha}\right),$$

where $k_n\equiv k$.

Proof. Let $h_j(t)=g_j^{(\alpha_j)}(t)$, $j=1,2,\cdots,n$ and $H_{\alpha j}(k)$ be the transform of $h_j(t)$. Then by using Theorem 2.4, we have

$$F_\alpha(k)=\sum_{k_{n-1}=0}^{k}\sum_{k_{n-2}=0}^{k_{n-1}}\cdots\sum_{k_2=0}^{k_3}\sum_{k_1=0}^{k_2} H_{\alpha 1}(k_1)H_{\alpha 2}(k_2-k_1)\cdots H_{\alpha(n-1)}(k_{n-1}-k_{n-2})H_{\alpha n}(k-k_{n-1}).$$

Under the conditions of Theorem 1.4 and using Theorem 2.5, we finally arrive at the required result.

Theorem 2.13. If $f(t)=g^{(-\beta)}(t)$, $m-1<\beta\leqslant m$, $m\in\mathbb{N}$ such that $\frac{\beta}{\alpha}\in\mathbb{Z}^+$ and the function $g(t)$ satisfies the conditions of Theorem 1.4, then

$$F_\alpha(k)=\frac{\Gamma(k\alpha-\beta+1)}{\Gamma(k\alpha+1)}G_\alpha(k-\beta/\alpha),$$

where $\alpha+1>\beta$, and $g^{(-\beta)}(t)$ represent the sequential Riemann-Liouville fractional integral operator defined in (1.1).

Proof. Using the fractional power series expansion, we have

$$f(t)=\mathcal{I}_a^\beta\sum_{k=0}^{\infty}G(k)(t-t_0)^{k\alpha}$$
$$=\sum_{k=0}^{\infty}G(k)(t-t_0)^{k\alpha+\beta}\frac{\Gamma(k\alpha+1)}{\Gamma(k\alpha+\beta+1)}$$

Starting the index of the series from $k=\beta/\alpha$,

$$f(t)=\sum_{k=\beta/\alpha}^{\infty} G(k-\beta/\alpha)(t-t_0)^{k\alpha}\frac{\Gamma(k\alpha-\beta+1)}{\Gamma(k\alpha+1)}$$

Now for $k>\beta/\alpha$ and by the definition of transform we get the required result.

Theorem 2.14. If $f(t)=\prod_{i=1}^{n} g_i^{(\alpha_i)}(t)\prod_{j=1}^{m} h_j^{(-\alpha_j)}(t)$, $m_i-1<\alpha_i\leqslant m_i$, $m_i\in\mathbb{N}$, $i=1,2,\cdots,n$ and the function $g_i(t)$ satisfies the conditions of Theorem 1.4, then

$$F_\alpha(k)=\sum_{l=0}^{k}\prod_{j=2}^{n}\sum_{k_{j-1}=0}^{k_j}\frac{\Gamma((k_j-k_{j-1})\alpha+\alpha_j+1)}{\Gamma((k_j-k_{j-1})\alpha+1)}$$

$$\times G_{\alpha j}\left(k_j-k_{j-1}+\frac{\alpha_j}{\alpha}\right)\prod_{i=2}^{m}\sum_{v_{i-1}=0}^{v_i}\frac{\Gamma((v_i-v_{i-1})\alpha-\alpha_i+1)}{\Gamma((v_i-v_{i-1})\alpha+1)}$$

$$\times H_{\alpha j}\left(v_i-v_{i-1}-\frac{\alpha_i}{\alpha}\right)H_{\alpha 1}\left(v_1-\frac{\alpha_1}{\alpha}\right)G_{\alpha 1}\left(k_1+\frac{\alpha_1}{\alpha}\right)$$

where $k_n\equiv k$ and $v_n\equiv k-l$.

Proof. Let $f(t)=g(t)h(t)$, where $g(t)=\prod_{i=1}^{n} g_i^{(\alpha_i)}(t)$, $h(t)=\prod_{j=1}^{m} h_j^{(-\alpha_j)}(t)$ and $G_\alpha(k)$, $H_\alpha(k)$ be the transform of $g(t)$, $h(t)$ respectively. Then by using Theorem 2.3, we have

$$F_\alpha(k)=\sum_{l=0}^{k} G_\alpha(k)H_\alpha(k-l).$$

Under the conditions of Theorem 1.4 and using Theorem 2.12-2.13, we finally arrive at the required result.

IV. NUMERICAL EXAMPLES

Example 3.1. Let us consider the linear fractional integro-differential equation

$$y^{(0.8)}(t)=\frac{24t^{3.2}}{\Gamma(4.2)}+\frac{48t^{3.6}}{\Gamma(4.6)}+\frac{120t^{4.1}}{\Gamma(5.1)}-2y^{(0.4)}-5y^{(-0.1)}, \tag{3.1}$$

with $y(0)=0$.

Setting $\alpha=0.1$ and using the Theorems 2.7 and 2.13 the equation (3.1) transforms to the following recurrence relation

$$Y(k+8)=\frac{\Gamma(0.1k+1)}{\Gamma(0.1k+1.8)}\left\{F(k)-2\frac{\Gamma(0.1k+1.4)}{\Gamma(0.1k+1)}Y(k+4)\right.$$
$$\left.-5\frac{\Gamma(0.1k+0.9)}{\Gamma(0.1k+1)}Y(k-1)\right\} \tag{3.2}$$

$$F(k)=\frac{24}{\Gamma(4.2)}\delta(k-32)+\frac{48}{\Gamma(4.6)}\delta(k-36)$$
$$+\frac{120}{\Gamma(5.1)}\delta(k-41) \tag{3.3}$$

Again by using (2.3) and initial condition, we get

$$Y(k)=0,\qquad k=0,1,\cdots,7. \tag{3.4}$$

Equation (3.2) and (3.3) are utilized to evaluate $Y(k)$ up to $N=40$ terms using Matlab and the numerical results and graph are presented in Table-1 and Figure-1 respectively. The exact solution to the problem (3.1) is $y=t^4$.

t	Exact	GDTM	Error
0	0	0	0
0.1	1.658E-3	1.658E-3	0
0.2	8.755E-3	8.757E-3	2.17E-20
0.3	2.317E-2	2.317E-2	0
0.4	4.621E-2	4.627E-2	6.939E-17
0.5	7.894E-2	7.901E-2	6.939E-17
0.6	1.222E-1	1.228E-1	5.551E-16
0.7	1.770E-1	1.775E-1	5.551E-16
0.8	2.434E-1	2.494E-1	5.551E-16
0.9	3.235E-1	3.602E-1	3.663E-15

Table 1

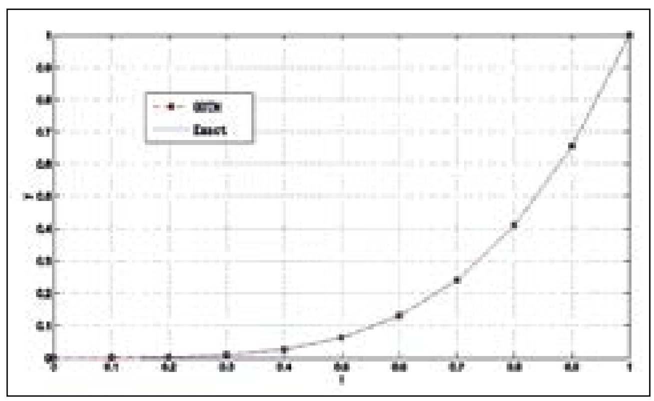

Figure-1

Example 3.2. Let us consider the linear fractional integro-differential equation

$$y^{(0.5)}(t)=(\sin t-\cos t))y^{(-p)}(t)+f(t)+t\int_0^t \sin x\, y(x)dx, \tag{3.5}$$

with $y(0)=0$ and

$$f(t)=\frac{2t^{1.5}}{\Gamma(2.5)}+\frac{t^{0.5}}{\Gamma(1.5)}+t(2-3\cos t-t\sin t+t^2\cos t). \tag{3.6}$$

Mittal and Nigam [16] solve the problem (3.5) by using Adomian decomposition method for the case $p=0$. Selecting the $\alpha = 0.1$, and using the Theorems 2.3, 2.6, 2.8 and 2.13 the equation (3.5) transforms to the following recurrence relations:

$$Y(k+5) = \frac{\Gamma(0.1k+1)}{\Gamma(0.1k+1.5)}\left\{\sum_{j=0}^{k}\frac{\Gamma((k-j)0.1-p+1)}{\Gamma((k-j)0.1+1)}\right.$$
$$\times(C(j)-S(j))Y(k-j-\beta/\alpha)+F(k)$$
$$\left.+\sum_{j=1/\alpha}^{k}\frac{\delta(k-j)}{j\alpha}\sum_{i=0}^{j-1/\alpha}S(i)Y(j-i-1/\alpha)\right\} \tag{3.7}$$

$$F(k) = \frac{2\delta(k-15)}{\Gamma(2.5)}+\frac{\delta(k-5)}{\Gamma(1.5)}+2\delta(k-10)$$
$$+\sum_{j=0}^{k}C(k-j)(\delta(k-30)-3\delta(k-10))$$
$$+\sum_{j=0}^{k}S(k-j)\delta(k-20). \tag{3.8}$$

where $C(k)$ and $S(k)$ represents the generalized differential transform for $\cos t$ and $\sin t$ respectively. Again by using (2.3) and the initial condition we get

$$Y(k)=0, \qquad k=0,1,\cdots,4. \tag{3.9}$$

The exact solution to the problem (3.5) is $y=t^2+t$. For $p=0,0.2,0.4$ the graph are presented in Figure-2.

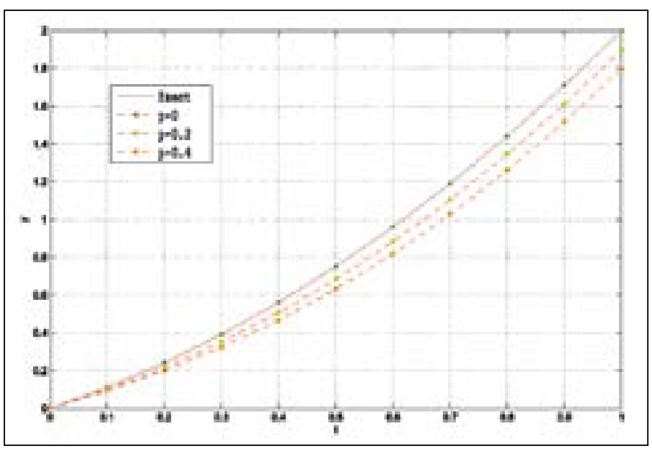

Figure-2

V. CONCLUSION

There were a few methods suggested to study the fractional integro-differential equations. The main concern of this article is to construct a numerical solution of the integro-differential equation of fractional order describe in Matter [14]. This generalized differential transform method is applicable to either initial or boundary value problems of fractional integro-differential equation which may be linear or nonlinear. It also possible to solve a system of fractional integro-differential equations by using this generalized method. It provides the solution in terms of convergent series with easily computable components and accuracy is improved by increasing number of terms considered. We have used the Math lab package to calculate the series obtained from the generalized differential transform method.

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DESIGN & ANALYSIS OF N-BIT COMPARATOR BASED ON LOW POWER ADIABATIC LOGIC TECHNIQUE

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ABSTRACT

This paper presents N-Bit Magnitude Comparator by Cascading 1-Bit Complex Cascadable Comparator based on low power adiabatic logic technique. At 0.3μm CMOS technology with L=0.3μm and W=0.75μm, the power consumptions is compared graphically at various frequencies with the counterpart conventional CMOS circuit using NI-Multisim. Two popular partially adiabatic circuits such as ECRL and PFAL are used as the reference circuits since they have got good improvement in power consumptions and mostly used as the reference circuit.

Index Terms -A.C power supply, energy recovery, adiabatic switching, Boolean expressions, power dissipations, waveforms and equivalent circuits.

I. NTRODUCTION

“Adiabatic” is a Greek word and used to describe the thermodynamic processes. which means no energy is exchange with environment (i.e no entropy enters or leaves the system) and therefore dissipated energy is almost zero.

Hence in adiabatic circuit the energy loss is being optimized. But the functional speed of the circuit is compromised since a.c or trapezoidal voltage source is used as inputs as well as supply voltage. In order to increase switching speed and decrease the area occupancy, the practical circuit is usually made up of an adiabatic component and a non-adiabatic component [1-3].

In conventional CMOS logic circuits (Fig.1), if an input is changed from 1 to 0 logic, the energy is transferred from the power supply to the output capacitor, the total charge is supply to the output node and the energy which is being drawn from the power supply is . But when the transition has ended, only half of the total energy is seen at the output load capacitor which is and the other half is lost in PMOS networks(F). From VDD to 0 transition of the output node, energy stored in the load capacitance is dissipated in the NMOS network (/F) [8].

Adiabatic logic circuits reduce the energy dissipation during switching process, and reuse the some of energy by recycling from the load capacitance [2].

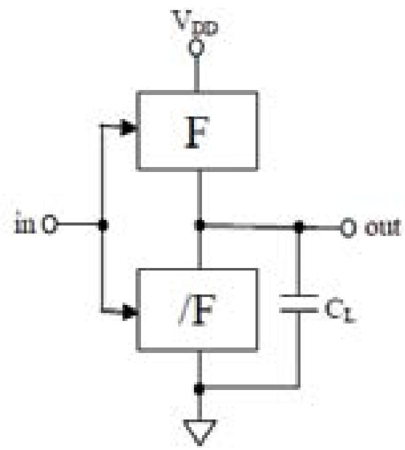

Fig.1: Conventional CMOS logic circuit with pull-up(F) and pull-down(/F) circuit [3].

II. CHARGING PROCESS IN ADIABATIC LOGIC CIRCUIT

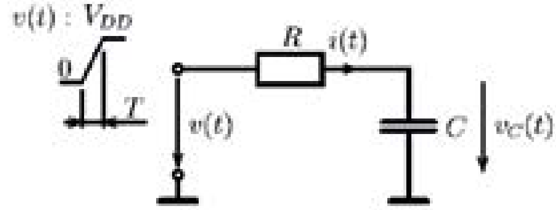

Fig.2: Adiabatic Charging

To calculate the energy consumed by charging a capacitance adiabatically, the equivalent circuit in Fig. 2 for an adiabatic gate is used. Here, the load capacitance C is charged by a constant current source. In conventional CMOS logic constant voltage source is used to charge the load capacitance.

Here, R is the on-resistance of PMOS network [9].

Therefore the current into the circuit can be determined by-

$$i(t) = \frac{Cdv(t)}{dt} = \frac{CV_{DD}}{T}$$

The energy for a charging event is calculated by integrating the power p(t) during the transition time T :

$$E = \int_0^T p(t)dt = \int_0^T v(t).i(t)dt$$

Or

$$E = \int_0^T (V_R(t) + V_c(t)).i(t)dt$$

Since no energy is dissipated in the capacitor at one clock cycle. Therefore energy expression becomes

$$E = \int_0^T RC^2 \frac{V_{DD}^{\ 2}}{T^2} \quad \text{Or} \quad E = \frac{RC}{T} CV_{DD}^{\ 2}$$

During recovery process the same amount of energy is wasted, Therefore the total energy dissipation over complete cycle is given as

$$E = \frac{2RC}{T} CV_{DD}^{\ 2}$$

From the above expression it can be concluded that the energy loss is inversely proportional to the switching time T. Here the interesting fact is that the energy consumption is not only govern by the time period T but also the resistance R which is absence in the conventional CMOS. Thus if T>>2RC then, the energy dissipation is lesser than the conventional CMOS [3, 10].

III. REFERENCE FAMILY USED

Practical adiabatic families can be classified as either partially adiabatic or fully adiabatic [11]. In a partially adiabatic circuit, some charge is allowed to be transferred to the ground, while in a fully adiabatic circuit, all the charge on the load capacitance is recovered by the power supply. Fully adiabatic circuits face problems with respect to the operating speed and the inputs power clock synchronization [1].

There are many adiabatic logic design techniques that are given in the literature. But here two of them are chosen, ECRL and PFAL which shows the good improvement in energy dissipation and are mostly used as reference in new logic families for less energy dissipation [2].

A. Efficient Charge Recovery Logic (ECRL)

It consists of two cross-coupled transistors M1 and M2 and two NMOS transistors in the N-functional blocks for the ECRL adiabatic logic block [12].

An AC power supply pwr is used for ECRL gates, so as to recover and reuse the supplied energy. Both out and /out are generated so that the power clock generator can always drive a constant load capacitance independent of the input signal [1].

Assuming ‘in’ is high and ‘/in’ is low, at the beginning of a cycle, when the clock ‘pwr’ rises, ‘out’ remains at a ground level, because ‘in ’ turn on M2. ‘/out’ follows ‘pwr through M1. When ‘pwr is high, the outputs hold valid logic levels. These values are used in the next stage for evaluation. While ‘pwr’ falls down to a ground level, charge on ‘/out’ returns its energy to ‘pwr’. Thus, the clock acts both as a clock and power supply [12].

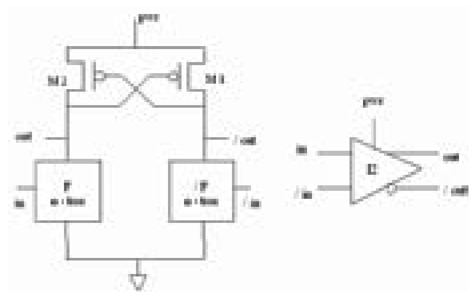

Fig.3: Basic model of ECRL circuit

B. Positive Feedback Adiabatic Logic (PFAL)

The partial energy recovery circuit structure so called Positive Feedback Adiabatic Logic (PFAL) has good robustness against technological parameter variations [8].

The core of all the PFAL gates is an adiabatic amplifier, a latch made by the two PMOS: M1-M2 and two NMOS: M3-M4, that avoids a logic level degradation on the output nodes out and /out. The two n-trees realize the logic functions. This logic family also generates both positive and negative outputs. The functional blocks are in parallel with the PMOSFETs of the adiabatic amplifier and form a transmission gate. The two n-trees realize the logic functions. This logic family also generates both positive and negative outputs [13].

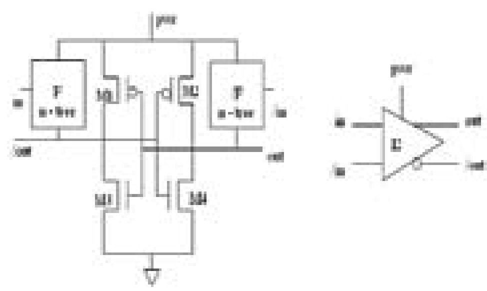

Fig.4: Basic model of PFAL circuit

IV. CIRCUIT IMPLEMENTATION

A. Formula and logical expression of the circuits

Comparators are designed to compare the magnitude of two bit binary numbers and indicate whether one is greater than, less than or equal to the other. An N-Bit comparator would accept N-Bit numbers and generate three outputs. The truth table of such 1-bit cascadable complex comparator is shown in Table 1. Here ai and bi are the ist stage of the inputs under comparison and xi, yi & zi are the previous stage of the outputs. The next stage of the outputs are denoted by xi+1, yi+1 and zi+1. The logical formulas for the next stage outputs are obtained from karnaugh maps [4].

Table1: Truth table of Cascadable 1-Bit Complex Comparator

S/N	ai	bi	xi	yi	zi	xi+1	yi+1	zi+1
0	0	0	0	0	0	-	-	-
1	0	0	0	0	1	0	0	1
2	0	0	0	1	0	0	1	1
3	0	0	0	1	1	-	-	-
4	0	0	1	0	0	1	0	0
5	0	0	1	0	1	-	-	-
6	0	0	1	1	0	-	-	-
7	0	0	1	1	1	-	-	-
8	0	1	0	0	0	-	-	-
9	0	1	0	0	1	0	0	1
10	0	1	0	1	0	0	0	1
11	0	1	0	1	1	-	-	-
12	0	1	1	0	0	0	0	1
13	0	1	1	0	1	-	-	-
14	0	1	1	1	0	-	-	-
15	0	1	1	1	1	-	-	-
16	1	0	0	0	0	-	-	-
17	1	0	0	0	1	1	0	0
18	1	0	0	1	0	1	0	0
19	1	0	0	1	1	-	-	-
20	1	0	1	0	0	1	0	0
21	1	0	1	0	1	-	-	-
22	1	0	1	1	0	-	-	-
23	1	0	1	1	1	-	-	-
24	1	1	0	0	0	-	-	-
25	1	1	0	0	1	0	0	1
26	1	1	0	1	0	0	1	0
27	1	1	0	1	1	-	-	-
28	1	1	1	0	0	1	0	0
29	1	1	1	0	1	-	-	-
30	1	1	1	1	0	-	-	-
31	1	1	1	1	1	-	-	-

Logical expression:

$$x_{i+1}(A > B) = a_i\overline{b}_i + a_i x_i + \overline{b}_i x_i$$

$$y_{i+1}(A = B) = \overline{a}_i\overline{b}_i y_i + a_i b_i y_i$$

$$z_{i+1}(A < B) = \overline{a}_i b_i + \overline{a}_i z_i + b_i z_i$$

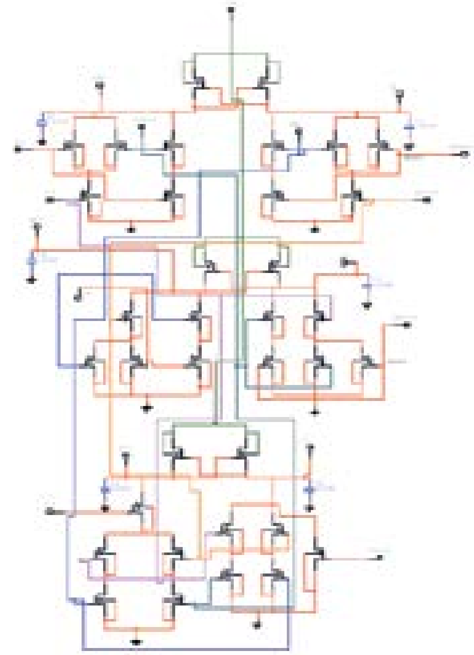

Fig.5: ECRL Cascadable 1-Bit Complex Comparator Circuit

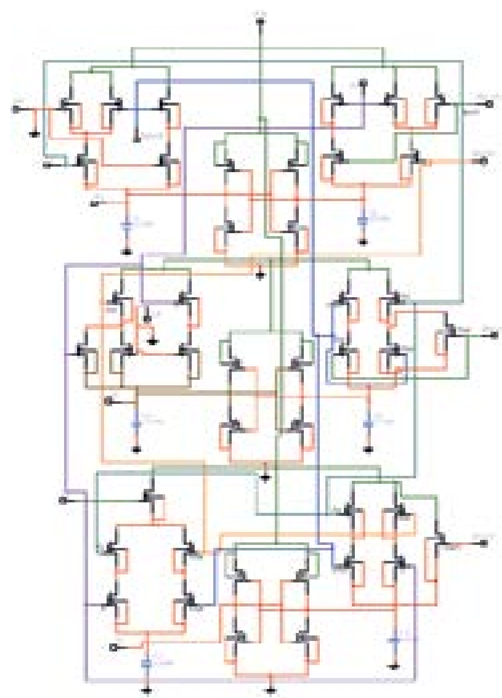

Fig.6:PFAL Cascadable 1-Bit Complex Comparator Circuit

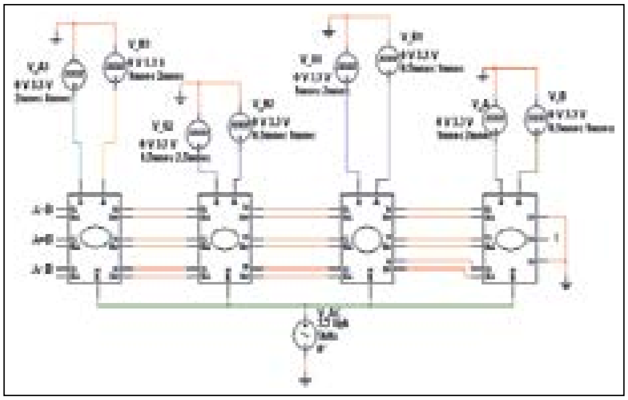

Fig.7: PFAL 4-Bit Magnitude Comparator Configured Using four Stages of Cascadable 1-Bit Complex Comparator.

B. Simulation Results

Simulations has been carried out in NI-Multisim. Fig.5 and Fig.6 shows ECRL and PFAL 1-Bit Cascadable Complex Comparator circuits. Fig.7 shows PFAL 4-Bit Magnitude Comparator by cascading 1-Bit Comparator.

The output waveforms of ECRL and PFAL are quite similar under same input conditions. Most often there is a requirement to compare two 4-bit, 8-bit or higher bit binary numbers. It is worthwhile to look at a 1-bit comparator with inputs and outputs which can be used to cascade several of these to configure multiple-bit comparators. Fig.8 and Fig.9 shows simulated waveforms of ECRL & PFAL 4-bit Magnitude comparator. Here the power clock has been indicated by PCK.

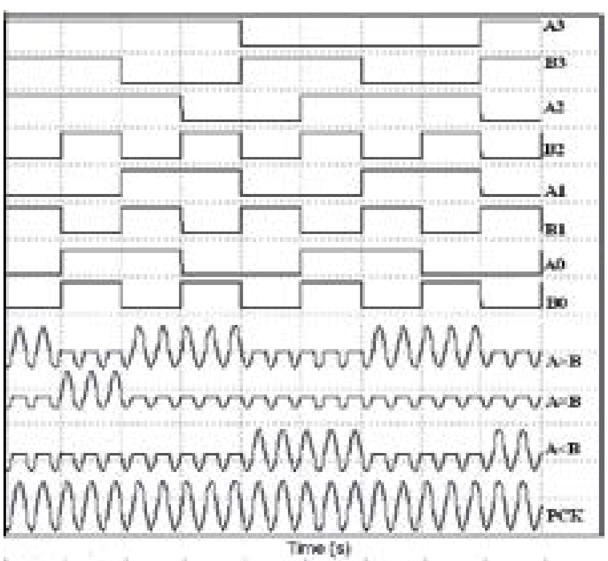

Fig.8: Simulated Waveform of the ECRL 4-Bit Magnitude Comparator by Cascading 1-Bit Complex Comparator

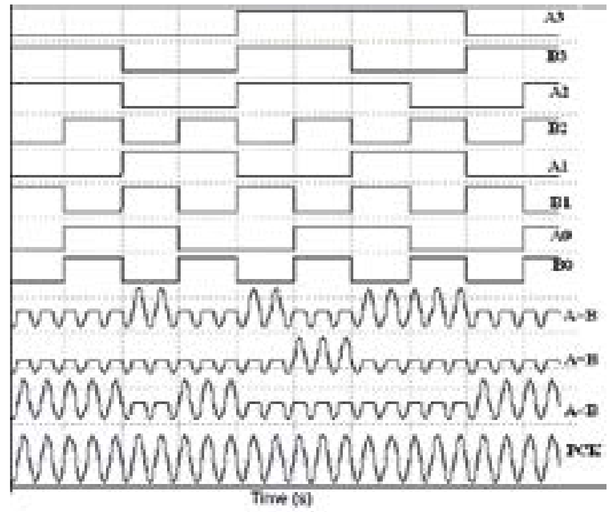

Fig.9: Simulated Waveform of the PFAL 4-Bit Magnitude Comparator by Cascading 1-Bit Complex Comparator

V. POWER CONSUMPTION ANALYSIS AND COMPARISON

Estimation of power consumptions is carried out at 0.3µm technology keeping the W/L ratio of the PMOS and NMOS are same and L=0.3µm and W=0.75µm is considered. The simulation has been done in NI-Multisim with load capacitance of 100fF at a frequency of 900Mhz. ECRL and PFAL logics are investigated against the conventional CMOS logic.

The graphical power analysis results of Cascadable 1-Bit Complex Comparator is shown in Fig.10. Table 2 and Table 3 Compares performances of PFAL, ECRL and traditional CMOS at two frequencies viz. 600mhz & 900Mhz in terms of transistor count, power dissipations and area consumption respectively [5] [6] [7].

Power dissipations of the circuits at different frequencies with same value of VDD=3.3V & CL=100fF are shown in Table 2 and Table 3.

Table 2: At 600MHZ:

CIRCUIT	Cascadable Complex 1-Bit Comparator		
	PFAL	ECRL	CMOS
Transistor count	42	36	32
Total power dissipation (µW)	1.9	2.3	7.45
Area per chip (µm2)	9.45	8.1	7.2

Table 3: At 900MHZ:

CIRCUIT	Cascadable Complex 1-Bit Comparator		
	PFAL	ECRL	CMOS
Transistor count	42	36	32
Total power dissipation (µW)	3.2	3.7	11.45
Area per chip (µm2)	9.45	8.1	7.2

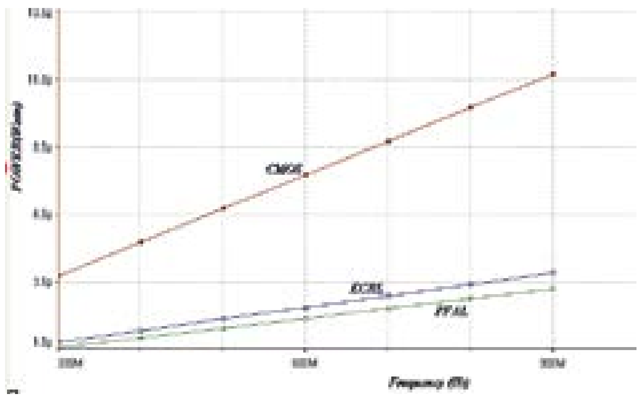

Fig.10: Simulated Power plot of Cascadable 1-Bit Complex Comparator

VI. CONCLUSION

In this paper we have presented Adiabatic N-Bit Magnitude Comparator circuits using PFAL and ECRL techniques which have better performance among the literature. For simplicity, here 4-Bit magnitude comparator is configured adiabatically by cascading four stages of 1-bit cascadable complex comparator. This shows that any multiple-bit could be compared adiabatically in the same pattern as desired. The circuit diagram and simulated output waveforms of both approaches are shown and the power dissipations of the circuit are evaluated at various frequencies and compared with the counterpart conventional CMOS circuits. From the above observations we have concluded that the design based on adiabatic principle gives superior performance when compared to traditional methods in terms of power even though their total area and transistor count is more in some circuits.

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ON SOME UNIFIED INTEGRALS

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ABSTRACT

In this paper, a number of integrals of various types involving the hypergeometric function, the H-function, the I-function are evaluated. First, we have evaluated certain finite integrals involving the hypergeometric function and H-function and then a few integrals involving product of the I-function with exponential function, hypergeometric function and H-function are evaluated. A number of particular cases of these integrals have also been recorded.

Key words: - Hypergeometric function; H-function; I-function.

I. NTRODUCTION

The Gaussian hypergeometric function is of fundamental importance in the theory of special functions. The importance of this function lies in the fact that almost all of the commonly used functions of applicable mathematics, mathematical physics, engineering and mathematical biology are expressible as its special cases. The series

$$ {}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} , \tag{1} $$

where $(a)_n$ is the Pochhammer symbol defined by

$$ (a)_n = \begin{cases} a(a+1)\cdots(a+n-1), & \text{if } \ n \in \mathrm{N} \\ 1, & \text{if } \ n = 0 \end{cases} , $$

is called the Gauss's hypergeometric series after the famous German mathematician Carl Friedrich Gauss (1777-1855) who in the year 1812 introduced this series. It is represented by the symbol ${}_2F_1(a,b;c;z)$ and is called the Gauss's hypergeometric function also.

In 1961, Charles Fox [2] introduced a function which is more general than the Meijer's G-function and this function is well known in the literature of special functions as Fox's H-function or simply the H-function. This function is defined and represented by means of the following Mellin-Barnes type contour integral:

$$ H_{p,q}^{m,n}\left[z \left| \begin{matrix} (a_j, e_j)_{1,p} \\ (b_j, f_j)_{1,q} \end{matrix} \right. \right] = \frac{1}{2\pi i} \int_L \theta(s)\, z^s ds , \tag{2} $$

where, for convenience,

$$ \theta(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - f_j s) \prod_{j=1}^{n} \Gamma(1 - a_j + e_j s)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + f_j s) \prod_{j=n+1}^{p} \Gamma(a_j - e_j s)} $$

and L is a suitable contour of the Mellin-Barnes type which runs from $\gamma - i\infty$ to $\gamma + i\infty$ (γ is real), separating the poles of $\Gamma(b_j - f_j s)$, $j = 1,\cdots,m$ from those of $\Gamma(1 - a_j + e_j s)$, $j = 1,\cdots,n$. An empty product is interpreted as unity. The integers m, n, p, q satisfy the inequalities $0 \le n \le p$, $0 \le m \le q$, the coefficients $e_j\ (j = 1,\cdots,p)$, $f_j\ (j = 1,\cdots,q)$ are positive real numbers, and the complex parameters $a_j (j = 1,\cdots,p)$, $b_j (j = 1,\cdots,q)$ are so constrained that no poles of the integrand coincide. Owing to the popularity of the special functions, those are defined in (1) and (2) (c.f. [4], [3] and [6]), details regarding these are avoided.

The *I*-function, which is more general than the Fox's *H*-function, defined by V.P. Saxena [5], by means of the following Mellin-Barnes type contour integral:

$$ I_{p_i,q_i:r}^{m,n}[z] = I_{p_i,q_i:r}^{m,n}\left[z \left| \begin{matrix} (a_j, \alpha_j)_{1,n} ; (a_{ji}, \alpha_{ji})_{n+1,p_i} \\ (b_j, \beta_j)_{1,m} ; (b_{ji}, \beta_{ji})_{m+1,q_i} \end{matrix} \right. \right] $$

$$ = \frac{1}{2\pi i} \int_L \phi(\xi)\, z^{\xi}\, d\xi \tag{3} $$

where,

$$ \phi(\xi) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j \xi) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j \xi)}{\sum_{i=1}^{r} \left\{ \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} + \beta_{ji} \xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji} - \alpha_{ji} \xi) \right\}} , $$

$p_i, q_i (i=1,\cdots,r), m, n$ are integers satisfying $0 \le n \le p_i$, $0 \le m \le q_i$; α_j, β_j, α_{ji}, β_{ji} are real and positive and a_j, b_j, a_{ji}, b_{ji} are complex numbers. L is a suitable contour of the Mellin-Barnes type running from $\gamma - i\infty$ to $\gamma + i\infty$ (γ is real) in the complex ξ-plane. Details regarding existence conditions and various parametric restrictions of *I*-function, we may refer [5].

For $r=1$, (2) reduces to the Fox's H-function

$$I^{m,n}_{p_i,q_i:1}\left[z \left| \begin{matrix} (a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right.\right] = H^{m,n}_{p,q}\left[z \left| \begin{matrix} (a_j,\alpha_j)_{1,n};(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m};(b_j,\beta_j)_{m+1,q} \end{matrix} \right.\right].$$

II. REQUIRED RESULTS

We shall require the following results in the sequel:

(i) [1, p. 399, eq. (4)]

For $\mathrm{Re}(\gamma) > 0$, $\mathrm{Re}(\rho) > 0$ and $\mathrm{Re}(\gamma+\rho-\alpha-\beta) > 0$

$$\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}\,{}_2F_1(\alpha,\beta;\gamma;x)dx = \frac{\Gamma(\gamma)\Gamma(\rho)\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\Gamma(\gamma+\rho-\beta)} \tag{4}$$

(ii) [4, p. 56, eq. (1)]

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} A(k,n) = \sum_{n=0}^{\infty}\sum_{k=0}^{n} A(k,n-k) \tag{5}$$

(iii) Mellin transform of the *H*-function [6, p. 15, eq. (2.4.1)]

$$\int_o^\infty x^{s-1} H^{m,n}_{p,q}\left[a x \left| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right] = a^{-s}\theta(-s)$$

$$= a^{-s}\frac{\prod_{j=1}^{m}\Gamma(b_j+\beta_j s)\ \prod_{j=1}^{n}\Gamma(1-a_j-\alpha_j s)}{\prod_{j=m+1}^{q}\Gamma(1-b_j-\beta_j s)\prod_{j=n+1}^{p}\Gamma(a_j+\alpha_j s)}, \tag{6}$$

where,

$$A = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p}\alpha_j + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q}\beta_j > 0$$

$$|\arg a| < \frac{1}{2}A\pi,\ \delta = \sum_{j=1}^{q}\beta_j - \sum_{j=1}^{p}\alpha_j > 0$$

and

$$-\min_{1\le j\le m}\left[\mathrm{Re}\left(b_j/\beta_j\right)\right] < \mathrm{Re}(s) < \min_{1\le j\le n}\left[\mathrm{Re}\left\{(1-a_j)/\alpha_j\right\}\right].$$

III. INTEGRALS INVOLVING THE HYERGEOMETRIC FUNCTION AND H-FUNCTION

In this section, we have evaluated certain finite integrals involving the hypergeometric function and *H*-function.

First Integral

$$I_1 \equiv \int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-xz}\,{}_2F_1(\alpha,\beta;\gamma;x)\,dx$$

$$= e^{-z}\frac{\Gamma(\gamma)\Gamma(\rho)\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\Gamma(\gamma+\rho-\beta)} \times {}_2F_2(\rho,\gamma+\rho-\alpha-\beta;\gamma+\rho-\alpha,\gamma+\rho-\beta;z) \tag{7}$$

where,

$\mathrm{Re}(\gamma) > 0$, $\mathrm{Re}(\rho) > 0$, $\mathrm{Re}(\gamma+\rho-\alpha-\beta) > 0$

Second Integral

$$I_2 \equiv \int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-xz}\,{}_2F_1(\alpha,\beta;\gamma;ax^{\zeta}) \times H^{m,n}_{p,q}\left[yx^{\mu}(1-x)^{\nu} \left| \begin{matrix} (a_j,e_j)_{1,p} \\ (b_j,f_j)_{1,q} \end{matrix} \right.\right] dx$$

$$= e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n} f(k)\frac{z^{u-k}}{(u-k)!} H^{m,n+2}_{p+2,q+1}\left[y \left| \begin{matrix} (1-\gamma-\zeta k,\mu), (1-\rho+k-u,\nu),(a_j,e_j)_{1,p} \\ (b_j,f_j)_{1,q}, (1-\gamma-\rho-(\zeta-1)k-u,\mu+\nu) \end{matrix} \right.\right], \tag{8}$$

where,

$$f(k) = \frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k}{k!}. \tag{9}$$

The conditions of validity of the integral in (8) are as follows:

(i) $\mu \ge 0$, $\nu \ge 0$ (not both zero simultaneously),

(ii) $|\arg y| < \frac{1}{2}A\pi$, where

$$A = \sum_{j=1}^{n} e_j - \sum_{j=n+1}^{p} e_j + \sum_{j=1}^{m} f_j - \sum_{j=m+1}^{q} f_j$$

(iii) $\mathrm{Re}(\gamma) + \mu \min_{1\le j\le m}\left[\mathrm{Re}\left(\frac{b_j}{f_j}\right)\right] > 0$, and

(iv) $\mathrm{Re}(\rho) + \nu \min_{1\le j\le m}\left[\mathrm{Re}\left(\frac{b_j}{f_j}\right)\right] > 0$

Third Integral

$$I_3 \equiv \int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-xz}\,{}_2F_1(\alpha,\beta;\gamma;ax^{\zeta})$$

$$\times H_{p,q}^{m,n}\left[\ yx^{-\mu}(1-x)^{-\nu}\left|\begin{matrix}(a_j,e_j)_{1,p}\\(b_j,f_j)_{1,q}\end{matrix}\right.\right]dx$$

$$=e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}$$

$$\times H_{p+1,q+2}^{m+2,n}\left[y\left|\begin{matrix}(a_j,e_j)_{1,p},(\gamma+\rho+(\zeta-1)k+u,\mu+\nu)\\(\gamma+\zeta k,\mu),(\rho-k+u,\nu),(b_j,f_j)_{1,q}\end{matrix}\right.\right] \tag{10}$$

Provided

$$\mathrm{Re}(\gamma)-\mu\max_{1\le j\le n}\left[\mathrm{Re}\left\{\tfrac{(a_j-1)}{e_j}\right\}\right]>0$$

$$\mathrm{Re}(\rho)-\nu\max_{1\le j\le n}\left[\mathrm{Re}\left\{\tfrac{(a_j-1)}{e_j}\right\}\right]>0$$

together with the conditions (i) and (ii) of (8) and $f(k)$ is given in (9).

Fourth Integral

$$I_4\equiv\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-xz}\,{}_2F_1(\alpha,\beta;\gamma;ax^{\zeta})$$

$$\times H_{p,q}^{m,n}\left[\ yx^{\mu}(1-x)^{-\nu}\left|\begin{matrix}(a_j,e_j)_{1,p}\\(b_j,f_j)_{1,q}\end{matrix}\right.\right]dx$$

$$=e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}$$

$$\times H_{p+1,q+2}^{m+1,n+1}\left[y\left|\begin{matrix}(1-\gamma-\zeta k,\mu),\ (a_j,e_j)_{1,p}\\(\rho-k+u,\nu),(b_j,f_j)_{1,q},\ (1-\gamma-\rho-(\zeta-1)k-u,\mu-\nu)\end{matrix}\right.\right], \tag{11}$$

Provided $\mu>0,\ \nu\ge 0$ such that $\mu-\nu\ge 0$ and

$$\mathrm{Re}(\gamma)+\mu\min_{1\le j\le m}\left[\mathrm{Re}\left(\tfrac{b_j}{f_j}\right)\right]>0$$

$$\mathrm{Re}(\rho)-\nu\max_{1\le j\le n}\left[\mathrm{Re}\left\{\tfrac{(a_j-1)}{e_j}\right\}\right]>0$$

together with the conditions (i) and (ii) of (8) and $f(k)$ is given in (9).

Fifth Integral

$$I_5\equiv\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{-xz}\,{}_2F_1(\alpha,\beta;\gamma;ax^{\zeta})$$

$$\times H_{p,q}^{m,n}\left[\ yx^{\mu}(1-x)^{-\nu}\left|\begin{matrix}(a_j,e_j)_{1,p}\\(b_j,f_j)_{1,q}\end{matrix}\right.\right]dx$$

$$=e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}$$

$$\times H_{p+2,q+1}^{m+1,n+1}\left[\ y\left|\begin{matrix}(1-\gamma-\zeta k,\mu),\ (a_j,e_j)_{1,p},(\gamma+\rho+(\zeta-1)k+u,\nu-\mu)\\(\rho-k+u,\nu),\ (b_j,f_j)_{1,q}\end{matrix}\right.\right], \tag{12}$$

Provided $\mu\ge 0,\ \nu>0$ such that $\nu-\mu\ge 0$ and

$$\mathrm{Re}(\gamma)-\mu\max_{1\le j\le n}\left[\mathrm{Re}\left\{\tfrac{(a_j-1)}{e_j}\right\}\right]>0$$

where, $\mu\ge 0,\ \nu>0$, such that $\nu-\mu\ge 0$ and

$$\mathrm{Re}(\gamma)-\mu\max_{1\le j\le n}\left[\mathrm{Re}\left\{\tfrac{(a_j-1)}{e_j}\right\}\right]>0$$

$$\mathrm{Re}(\rho)+\nu\min_{1\le j\le m}\left[\mathrm{Re}\left(\tfrac{b_j}{f_j}\right)\right]>0$$

together with the conditions (i) and (ii) of (8) and $f(k)$ is given in (9).

Proof of (7):

$$I_1\equiv e^{-z}\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{(1-x)z}\,{}_2F_1(\alpha,\beta;\gamma;x)\,dx$$

$$=e^{-z}\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}\sum_{r=0}^{\infty}\frac{(1-x)^r z^r}{r!}$$

$$\times{}_2F_1(\alpha,\beta;\gamma;x)\,dx$$

Now changing the order of integration and summation, we obtain

$$I_1=e^{-z}\left\{\sum_{r=0}^{\infty}\frac{z^r}{r!}\int_0^1 x^{\gamma-1}(1-x)^{\rho+r-1}\times{}_2F_1(\alpha,\beta;\gamma;x)\,dx\right\}$$

Evaluating the integral with the help of (4), we get

$$I_1=e^{-z}\sum_{r=0}^{\infty}\frac{z^r}{r!}\frac{\Gamma(\gamma)\Gamma(\rho+r)\Gamma(\gamma+\rho+r-\alpha-\beta)}{\Gamma(\gamma+\rho+r-\alpha)\Gamma(\gamma+\rho+r-\beta)}$$

$$=e^{-z}\left[\frac{\Gamma(\gamma)\Gamma(\rho)\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\Gamma(\gamma+\rho-\beta)}+\frac{\Gamma(\gamma)\Gamma(\rho+1)\Gamma(\gamma+\rho-\alpha-\beta+1)}{\Gamma(\gamma+\rho-\alpha+1)\Gamma(\gamma+\rho-\beta+1)}z+\frac{\Gamma(\gamma)\Gamma(\rho+2)\Gamma(\gamma+\rho-\alpha-\beta+2)}{\Gamma(\gamma+\rho-\alpha+2)\Gamma(\gamma+\rho-\beta+2)}\frac{z^2}{2!}+\cdots\right]$$

$$= e^{-z}\frac{\Gamma(\gamma)\Gamma(\rho)\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\Gamma(\gamma+\rho-\beta)}\Bigg[\ 1+$$

$$\frac{\rho(\gamma+\rho-\alpha-\beta)}{(\gamma+\rho-\alpha)(\gamma+\rho-\beta)}z+\frac{\rho(\rho+1)}{(\gamma+\rho-\alpha)}$$

$$\frac{(\gamma+\rho-\alpha-\beta)(\gamma+\rho-\alpha-\beta+1)}{(\gamma+\rho-\alpha+1)(\gamma+\rho-\beta+1)}\frac{z^2}{2!}+\cdots\Bigg]$$

$$= e^{-z}\frac{\Gamma(\gamma)\Gamma(\rho)\Gamma(\gamma+\rho-\alpha-\beta)}{\Gamma(\gamma+\rho-\alpha)\Gamma(\gamma+\rho-\beta)}$$

$$\times {}_2F_2(\rho,\gamma+\rho-\alpha-\beta;\gamma+\rho-\alpha,\gamma+\rho-\beta;z)$$

Proof of (8):

$$I_2 \equiv e^{-z}\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}e^{(1-x)z}\,{}_2F_1(\alpha,\beta;\gamma;a\,x^{\zeta})$$

$$\times H_{p,q}^{m,n}\left[\ yx^{\mu}(1-x)^{\nu}\left|\begin{matrix}(a_j,e_j)_{1,p}\\(b_j,f_j)_{1,q}\end{matrix}\right.\right]dx$$

Now we replace $e^{(1-x)z}$ by $\sum_{u=0}^{\infty}\frac{(1-x)^u z^u}{u!}$ and express the hypergeometric function and the H function with the help of (1) and (2) respectively, to get

$$I_2 = e^{-z}\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}\sum_{u=0}^{\infty}\frac{(1-x)^u z^u}{u!}$$

$$\times\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\zeta k}}{k!}$$

$$\times\frac{1}{2\pi i}\int_L \theta(s)y^s x^{\mu s}(1-x)^{\nu s}\,ds\,dx$$

$$= e^{-z}\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}\sum_{u=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\zeta k}}{k!}$$

$$\times\frac{(1-x)^u z^u}{u!}\frac{1}{2\pi i}\int_L \theta(s)y^s x^{\mu s}(1-x)^{\nu s}\,ds\,dx$$

By the use of (5), the above result reduces to

$$I_2 = e^{-z}\int_0^1 x^{\gamma-1}(1-x)^{\rho-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\zeta k}}{k!}$$

$$\times\frac{(1-x)^{u-k}z^{u-k}}{(u-k)!}\frac{1}{2\pi i}\int_L \theta(s)y^s x^{\mu s}(1-x)^{\nu s}\,ds\,dx$$

Interchanging the order of integration and summation, we obtain

$$I_2 = e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}\frac{1}{2\pi i}\int_L \theta(s)y^s$$

$$\times\left\{\int_0^1 x^{\gamma+\zeta k+\mu s-1}(1-x)^{\rho+u-k+\nu s-1}dx\right\}ds$$

$$= e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}$$

$$\times\frac{1}{2\pi i}\int_L \theta(s)\frac{\Gamma(\gamma+\zeta k+\mu s)\,\Gamma(\rho+u-k+\nu s)}{\Gamma(\gamma+\rho+(\zeta-1)k+u+(\mu+\nu)s)}y^s ds$$

where, $f(k)=\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k}{k!}$.

Finally, interpreting the contour integral by virtue of (2), we obtain

$$I_2 = e^{-z}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}H_{p+2,q+1}^{m,n+2}$$

$$\left[y\left|\begin{matrix}(1-\gamma-\zeta k,\mu),(1-\rho+k-u,\nu),(a_j,e_j)_{1,p}\\(b_j,f_j)_{1,q}\ ,(1-\gamma-\rho-(\zeta-1)k-u,\mu+\nu)\end{matrix}\right.\right]$$

The integrals (10) to (14) can be proved on lines similar to those of integral (8).

IV. INTEGRALS INVOLVING *I*-FUNCTION

In this section, we have evaluated certain integrals involving product of the I-function with exponential function, hypergeometric function and H-function.

Eighth Integral

$$I_8 \equiv \int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}F_1\left(\alpha,\beta;\gamma;a\,x^{\zeta}(t-x)^{\eta}\right)$$

$$\times I_{p_i,q_i:r}^{m,n}\left[yx^{\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$= e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I_{p_i+2,q_i+1:r}^{m,n+2}\left[\ yt^{\mu+\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(1-\sigma-(\eta-1)k-u,\nu\right),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\\left(1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu+\nu\right)\end{matrix}\right], \tag{15}$$

where,

$$f(k)=\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k}{k!}, \tag{16}$$

Provided

(i) $\mu\geq 0$, $\nu\geq 0$ (not both zero simultaneously),

(ii) ζ and η are non-negative integers such that $\zeta+\eta\geq 1$

(iii) $A_i>0$, $B_i<0$; $|\arg y|<\frac{1}{2}A_i\pi$,

$\forall i\in 1,\cdots,r$; where

$$A_i=\sum_{j=1}^{n}\alpha_j-\sum_{j=n+1}^{p_i}\alpha_{ji}+\sum_{j=1}^{m}\beta_j-\sum_{j=m+1}^{q_i}\beta_{ji}$$

$$B_i=\frac{1}{2}(p_i-q_i)+\sum_{j=1}^{q_i}b_{ji}-\sum_{j=1}^{p_i}a_{ji}$$

(iv) $\operatorname{Re}(\rho)+\mu\min\limits_{1\leq j\leq m}\left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right]>0$,

$\operatorname{Re}(\sigma)+\nu\min\limits_{1\leq j\leq m}\left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right]>0$.

Ninth Integral

$$I_9\equiv\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}\,{}_2F_1\left(\alpha,\beta;\gamma;ax^{\zeta}(t-x)^{\eta}\right)$$

$$\times I_{p_i,q_i:r}^{m,n}\left[yx^{-\mu}(t-x)^{-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I_{p_i+1,q_i+2:r}^{m+2,n}\left[yt^{-\mu-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};\\(\rho+\zeta k,\mu),\end{matrix}\right.\right.$$

$$\left.\left.\begin{matrix}(a_{ji},\alpha_{ji})_{n+1,p_i},(\rho+\sigma+(\zeta+\eta-1)k+u,\mu+\nu)\\(\sigma+(\eta-1)k+u,\nu),(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right],\qquad(17)$$

Provided

$\operatorname{Re}(\rho)-\mu\max\limits_{1\leq j\leq n}\left[\operatorname{Re}\left\{{}^{(a_j-1)}\!/_{\alpha_j}\right\}\right]>0$,

$\operatorname{Re}(\sigma)-\nu\max\limits_{1\leq j\leq n}\left[\operatorname{Re}\left\{{}^{(a_j-1)}\!/_{\alpha_j}\right\}\right]>0$,

along with the sets of conditions (i) to (iii) given with I_8 and $f(k)$ is given by (16).

Tenth Integral

$$I_{10}\equiv\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}\,{}_2F_1\left(\alpha,\beta;\gamma;ax^{\zeta}(t-x)^{\eta}\right)$$

$$\times I_{p_i,q_i:r}^{m,n}\left[yx^{\mu}(t-x)^{-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$I_{p_i+1,q_i+2:r}^{m+1,n+1}\left[yt^{\mu-\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),(a_j,\alpha_j)_{1,n};\\(\sigma+(\eta-1)k+u,\nu),(b_j,\beta_j)_{1,m};\end{matrix}\right.\right.$$

$$\left.\left.\begin{matrix}(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_{ji},\beta_{ji})_{m+1,q_i},(1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu-\nu)\end{matrix}\right.\right],\qquad(18)$$

Provided $\mu>0$, $\nu\geq 0$ such that $\mu-\nu\geq 0$,

$\operatorname{Re}(\rho)+\mu\min\limits_{1\leq j\leq m}\left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right]>0$,

$\operatorname{Re}(\sigma)-\nu\max\limits_{1\leq j\leq n}\left[\operatorname{Re}\left\{{}^{(a_j-1)}\!/_{\alpha_j}\right\}\right]>0$,

along with the sets of conditions (i) to (iii) given with I_8 and $f(k)$ is given by (16).

Eleventh Integral

$$I_{11}\equiv\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}\,{}_2F_1\left(\alpha,\beta;\gamma;ax^{\zeta}(t-x)^{\eta}\right)$$

$$\times I_{p_i,q_i:r}^{m,n}\left[yx^{\mu}(t-x)^{-\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I_{p_i+2,q_i+1:r}^{m+1,n+1}\left[yt^{\mu-\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),(a_j,\alpha_j)_{1,n};\\(\sigma+(\eta-1)k+u,\nu),(b_j,\beta_j)_{1,m};\end{matrix}\right.\right.$$

$$\left.\left.\begin{matrix}(a_{ji},\alpha_{ji})_{n+1,p_i},(\rho+\sigma+(\zeta+\eta-1)k+u,\nu-\mu)\\(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right],\qquad(19)$$

Provided $\mu\geq 0$, $\nu>0$ such that $\nu-\mu\geq 0$,

$\operatorname{Re}(\rho)-\mu\max\limits_{1\leq j\leq n}\left[\operatorname{Re}\left\{{}^{(a_j-1)}\!/_{\alpha_j}\right\}\right]>0$,

$\operatorname{Re}(\sigma)+\nu\min\limits_{1\leq j\leq m}\left[\operatorname{Re}\left(\frac{b_j}{\beta_j}\right)\right]>0$,

along with the sets of conditions (i) to (iii) given with I_8 and $f(k)$ is given by (16).

Twelfth Integral

$$I_{12}\equiv\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}\,{}_2F_1\left(\alpha,\beta;\gamma;ax^{\zeta}(t-x)^{\eta}\right)$$

$$\times I^{m,n}_{p_i,q_i:r}\left[yx^{-\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m+1,n+1}_{p_i+2,q_i+1:r}\left[\;y\,t^{-\mu+\nu}\left|\begin{matrix}(1-\sigma-(\eta-1)k-u,\nu),(a_j,\alpha_j)_{1,n};\\(\rho+\zeta k,\mu),(b_j,\beta_j)_{1,m};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_{ji},\alpha_{ji})_{n+1,p_i},(\rho+\sigma+(\zeta+\eta-1)k+u,\mu-\nu)\\(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right], \tag{20}$$

Provided $\mu>0,\ \nu\geq 0$ such that $\mu-\nu\geq 0$,

$$\mathrm{Re}(\rho)+\mu\min_{1\leq j\leq m}\left[\mathrm{Re}\left(\tfrac{b_j}{\beta_j}\right)\right]>0,$$

$$\mathrm{Re}(\sigma)-\nu\max_{1\leq j\leq n}\left[\mathrm{Re}\left\{{}^{(a_j-1)}\!/_{\alpha_j}\right\}\right]>0,$$

along with the sets of conditions (i) to (iii) given with I_8 and $f(k)$ is given by (16).

Thirteenth Integral

$$I_{13}\equiv\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{-xz}\,{}_2F_1\left(\alpha,\beta;\gamma;a\,x^{\zeta}(t-x)^{\eta}\right)$$

$$\times I^{m,n}_{p_i,q_i:r}\left[yx^{-\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

$$=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I^{m+1,n+1}_{p_i+1,q_i+2:r}y\,t^{-\mu+\nu}\left[\;\left|\begin{matrix}\left(1-\sigma-(\eta-1)k-u,\nu\right),\\(\rho+\zeta k,\mu),(b_j,\beta_j)_{1,m};\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_{ji},\beta_{ji})_{m+1,q_i},(1-\rho-\sigma-(\zeta+\eta-1)k-u,\nu-\mu)\end{matrix}\right], \tag{21}$$

Provided $\mu\geq 0,\ \nu>0$ such that $\nu-\mu\geq 0$,

$$\mathrm{Re}(\rho)-\mu\max_{1\leq j\leq n}\left[\mathrm{Re}\left\{{}^{(a_j-1)}\!/_{\alpha_j}\right\}\right]>0,$$

$$\mathrm{Re}(\sigma)+\nu\min_{1\leq j\leq m}\left[\mathrm{Re}\left(\tfrac{b_j}{\beta_j}\right)\right]>0,$$

along with the sets of conditions (i) to (iii) given with I_8 and $f(k)$ is given by (16).

Fourteenth Integral

$$I_{14}\equiv\int_0^{\infty}x^{\eta-1}e^{ax}\,{}_2F_1(\alpha,\beta;\gamma;a\,x^{\rho})$$

$$\times I^{m_1,n_1}_{p_i,q_i:r}\left[z\,x^{\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,n_1};(a_{ji},\alpha_{ji})_{n_1+1,p_i}\\(b_j,\beta_j)_{1,m_1};(b_{ji},\beta_{ji})_{m_1+1,q_i}\end{matrix}\right.\right]$$

$$\times H^{m,n}_{p,q}\left[w\,x\left|\begin{matrix}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m};(d_j,\delta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=w^{-\eta}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{a^{u-k}}{(u-k)!}w^{-(\rho-1)k-u}$$

$$\times I^{m_1+n,n_1+m}_{p_i+q,q_i+p:r}\left[\;z\,w^{-\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,n_1},\\(b_j,\beta_j)_{1,m_1},\end{matrix}\right.\right.$$

$$\left(1-d_j-(\eta+(\rho-1)k+u)\delta_j,\sigma\delta_j\right)_{1,m};$$

$$\left(1-c_j-(\eta+(\rho-1)k+u)\gamma_j,\sigma\gamma_j\right)_{1,n};$$

$$(a_{ji},\alpha_{ji})_{n_1+1,p_i},$$

$$(b_{ji},\beta_{ji})_{m_1+1,q_i},$$

$$\left.\begin{matrix}\left(1-d_j-(\eta+(\rho-1)k+u)\delta_j,\sigma\delta_j\right)_{m+1,q}\\\left(1-c_j-(\eta+(\rho-1)k+u)\gamma_j,\sigma\gamma_j\right)_{n+1,p}\end{matrix}\right], \tag{22}$$

where,

$$f(k)=\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k}{k!} \tag{23}$$

Provided

(i) $\lambda>0,\ |\arg z|<\frac{1}{2}\pi\lambda$

(ii) $\lambda\geq 0,\ |\arg z|\leq\frac{1}{2}\pi\lambda,\ \mathrm{Re}(\mu+1)<0$

(iii) $\lambda_1>0,\ |\arg w|<\frac{1}{2}\pi\lambda_1$

(iv) $\lambda_1\geq 0,\ |\arg w|\leq\frac{1}{2}\pi\lambda_1,\ \mathrm{Re}(\mu_1+1)<0$

(v) $\sigma>0$,

$$-\sigma\min_{1\leq j\leq m_1}\left[\mathrm{Re}\left(\tfrac{b_j}{\beta_j}\right)\right]-\min_{1\leq j\leq m}\left[\mathrm{Re}\left(\tfrac{d_j}{\delta_j}\right)\right]$$

$$<\mathrm{Re}(\eta)<\sigma<\min_{1\leq j\leq n_1}\left[\mathrm{Re}\left\{(1-a_j)/\alpha_j\right\}\right]$$

$$+\min_{1\leq j\leq n}\left[\mathrm{Re}\left\{(1-c_j)/\gamma_j\right\}\right]$$

where,

$$\lambda=\sum_{j=1}^{n_1}\alpha_j+\sum_{j=1}^{m_1}\beta_j-\max_{1\le i\le r}\left[\sum_{j=n_1+1}^{p_i}\alpha_{ji}+\sum_{j=m_1+1}^{q_i}\beta_{ji}\right]$$

$$\mu=\sum_{j=1}^{m_1}b_j-\sum_{j=1}^{n_1}a_j-\min_{1\le i\le r}\left[\sum_{j=n_1+1}^{p_i}a_{ji}\right.$$

$$\left.-\sum_{j=m_1+1}^{q_i}b_{ji}+\frac{1}{2}(p_i-q_i)\right]$$

$$\lambda_1=\sum_{j=1}^{m}\delta_j+\sum_{j=1}^{n}\gamma_j-\sum_{j=m+1}^{q}\delta_j-\sum_{j=n+1}^{p}\gamma_j$$

$$\mu_1=\frac{1}{2}(p-q)+\sum_{j=1}^{q}d_j-\sum_{j=1}^{p}c_j$$

Proof of (15):

$$I_8\equiv e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}e^{(t-x)z}$$

$$\times{}_2F_1\left(\alpha,\beta;\gamma;a\,x^{\zeta}(t-x)^{\eta}\right)$$

$$\times I_{p_i,q_i:r}^{m,n}\left[yx^{\mu}(t-x)^{\nu}\left|\begin{matrix}(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx$$

Now we replace $e^{(t-x)z}$ by $\sum_{u=0}^{\infty}\frac{(t-x)^u z^u}{u!}$ and express the hypergeometric function and the *I*-function with the help of (1) and (3) respectively, to get

$$I_8=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\frac{(t-x)^u z^u}{u!}$$

$$\times\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\zeta k}(t-x)^{\eta k}}{k!}$$

$$\times\frac{1}{2\pi i}\int_L\phi(\xi)y^{\xi}x^{\mu\xi}(t-x)^{\nu\xi}d\xi\,dx$$

$$=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}$$

$$\times\frac{a^k x^{\zeta k}(t-x)^{\eta k+u}}{k!}\frac{z^u}{u!}$$

$$\times\frac{1}{2\pi i}\int_L\phi(\xi)y^{\xi}x^{\mu\xi}(t-x)^{\nu\xi}d\xi\,dx$$

Now by the use of (5), the above result reduces to

$$I_8=e^{-zt}\int_0^t x^{\rho-1}(t-x)^{\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}$$

$$\times\frac{a^k x^{\zeta k}(t-x)^{\eta k+u-k}}{k!}\frac{z^{u-k}}{(u-k)!}$$

$$\times\frac{1}{2\pi i}\int_L\phi(\xi)y^{\xi}x^{\mu\xi}(t-x)^{\nu\xi}d\xi\,dx$$

Interchanging the order of integration and summation, we obtain

$$I_{10}=e^{-zt}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}\frac{1}{2\pi i}\int_L\phi(\xi)y^{\xi}$$

$$\times\left\{\int_0^t x^{\rho+\zeta k+\mu\xi-1}(t-x)^{\sigma+(\eta-1)k+u+\nu\xi-1}dx\right\}d\xi,$$

where $f(k)$ is given in (16).

On substituting $x=ts$ in the inner *x*-integral, the above expression reduces to

$$I_8=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times\frac{1}{2\pi i}\int_L\phi(\xi)y^{\xi}t^{(\mu+\nu)\xi}$$

$$\times\left\{\int_0^1 s^{\rho+\zeta k+\mu\xi-1}(1-s)^{\sigma+(\eta-1)k+u+\nu\xi-1}ds\right\}d\xi$$

$$=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}\frac{1}{2\pi i}$$

$$\times\int_L\phi(\xi)\frac{\Gamma(\rho+\zeta k+\mu\xi)\Gamma\left(\sigma+(\eta-1)k+u+\nu\xi\right)}{\Gamma\left(\rho+\sigma+(\zeta+\eta-1)k+u+(\mu+\nu)\xi\right)}$$

$$\times y^{\xi}t^{(\mu+\nu)\xi}d\xi$$

Finally, interpreting the contour integral by virtue of (3), we obtain

$$I_8=e^{-zt}t^{\rho+\sigma-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}f(k)\frac{z^{u-k}}{(u-k)!}t^{(\zeta+\eta-1)k+u}$$

$$\times I_{p_i+2,q_i+1:r}^{m,n+2}\left[\;y\,t^{\mu+\nu}\left|\begin{matrix}(1-\rho-\zeta k,\mu),\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i},\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\left(1-\sigma-(\eta-1)k-u,\nu\right),(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\\left(1-\rho-\sigma-(\zeta+\eta-1)k-u,\mu+\nu\right)\end{matrix}\right].$$

The integrals (17) to (21) can be proved on lines similar to those of integral (15).

Proof of (22):

We replace e^{ax} by $\sum_{u=0}^{\infty}\frac{a^u x^u}{u!}$ and express the hypergeometric function and the *I*-function with the help of (1) and (3) respectively, to obtain

$$I_{14}=\int_0^\infty x^{\eta-1}\sum_{u=0}^{\infty}\frac{a^u x^u}{u!}\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\rho k}}{k!}$$

$$\times\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} x^{\sigma\xi}$$

$$\times H^{m,n}_{p,q}\left[wx\left|\begin{matrix}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m};(d_j,\delta_j)_{m+1,q}\end{matrix}\right.\right]d\xi\,dx$$

$$=\int_0^\infty x^{\eta-1}\sum_{u=0}^{\infty}\sum_{k=0}^{\infty}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\rho k+u}}{k!}\frac{a^u}{u!}$$

$$\times\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} x^{\sigma\xi}$$

$$\times H^{m,n}_{p,q}\left[wx\left|\begin{matrix}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m};(d_j,\delta_j)_{m+1,q}\end{matrix}\right.\right]d\xi\,dx$$

Now by the use of (5), the above result reduces to

$$I_{14}=\int_0^\infty x^{\eta-1}\sum_{u=0}^{\infty}\sum_{k=0}^{n}\frac{(\alpha)_k(\beta)_k}{(\gamma)_k}\frac{a^k x^{\rho k+u-k}}{k!}\frac{a^{u-k}}{u-k!}$$

$$\times\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} x^{\sigma\xi}$$

$$\times H^{m,n}_{p,q}\left[wx\left|\begin{matrix}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m};(d_j,\delta_j)_{m+1,q}\end{matrix}\right.\right]d\xi\,dx$$

Interchanging the order of integration and summation, we obtain

$$I_{14}=\sum_{u=0}^{\infty}\sum_{k=0}^{n} f(k)\frac{a^{u-k}}{(u-k)!}$$

$$\times\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi}\left\{\int_0^\infty x^{\eta+(\rho-1)k+u+\sigma\xi-1}\right.$$

$$\left.\times H^{m,n}_{p,q}\left[wx\left|\begin{matrix}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\(d_j,\delta_j)_{1,m};(d_j,\delta_j)_{m+1,q}\end{matrix}\right.\right]dx\right\}d\xi,$$ where

$f(k)$ is given by (23).

Now we use the Mellin transform of *H*-function by virtue of (5), so that

$$I_{14}=\sum_{u=0}^{\infty}\sum_{k=0}^{n} f(k)\frac{a^{u-k}}{(u-k)!}$$

$$\times\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} w^{-(\eta+(\rho-1)k+u+\sigma\xi)}$$

$$\times\frac{\prod_{j=1}^{m}\Gamma\left(d_j+\delta_j(\eta+(\rho-1)k+u+\sigma\xi)\right)}{\prod_{j=m+1}^{q}\Gamma\left(1-d_j-\delta_j(\eta+(\rho-1)k+u+\sigma\xi)\right)}$$

$$\times\frac{\prod_{j=1}^{n}\Gamma\left(1-c_j-\gamma_j(\eta+(\rho-1)k+u+\sigma\xi)\right)}{\prod_{j=n+1}^{p}\Gamma\left(c_j+\gamma_j(\eta+(\rho-1)k+u+\sigma\xi)\right)}d\xi$$

$$=w^{-\eta}\sum_{u=0}^{\infty}\sum_{k=0}^{n} f(k)\frac{a^{u-k}}{(u-k)!}w^{-(\rho-1)k-u}\frac{1}{2\pi i}\int_L \phi(\xi)$$

$$\times\frac{\prod_{j=1}^{m}\Gamma\left(d_j+(\eta+(\rho-1)k+u)\delta_j+\sigma\delta_j\xi\right)}{\prod_{j=m+1}^{q}\Gamma\left(1-d_j-(\eta+(\rho-1)k+u)\delta_j-\sigma\delta_j\xi\right)}$$

$$\times\frac{\prod_{j=1}^{n}\Gamma\left(1-c_j-(\eta+(\rho-1)k+u)\gamma_j-\sigma\gamma_j\xi\right)}{\prod_{j=n+1}^{p}\Gamma\left(c_j+(\eta+(\rho-1)k+u)\gamma_j+\sigma\gamma_j\xi\right)}$$

$$\times z^{\xi} w^{-\sigma\xi}d\xi$$

Finally, interpreting the contour integral by virtue of (3), we obtain

$$I_{14}=w^{-\eta}\sum_{u=}^{\infty}\sum_{k=0}^{n} f(k)\frac{a^{u-k}}{(u-k)!}w^{-(\rho-1)k-u}$$

$$\times I^{m_1+n,n_1+m}_{p_i+q,q_i+p:r}\left[zw^{-\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,n_1},\\(b_j,\beta_j)_{1,m_1},\end{matrix}\right.\right.$$

$$\left(1-d_j-(\eta+(\rho-1)k+u)\delta_j,\sigma\delta_j\right)_{1,m};$$

$$\left(1-c_j-(\eta+(\rho-1)k+u)\gamma_j,\sigma\gamma_j\right)_{1,n};$$

$$(a_{ji},\alpha_{ji})_{n_1+1,p_i},$$

$$(b_{ji},\beta_{ji})_{m_1+1,q_i},$$

$$\left.\begin{matrix}\left(1-d_j-(\eta+(\rho-1)k+u)\delta_j,\sigma\delta_j\right)_{m+1,q}\\\left(1-c_j-(\eta+(\rho-1)k+u)\gamma_j,\sigma\gamma_j\right)_{n+1,p}\end{matrix}\right]$$

2.5 Particular Cases:

(i) Integrals (8), (10), (11), (12), (13) and (14) are the particular cases of the integrals (15), (17), (18), (19), (20) and

(21), respectively, on putting $r=1$, $t=1$, and $\eta=0$ in them.

(iv) Putting $a=0$ in (22), the exponential function e^{ax} and the hypergeometric function reduces to unity and consequently it leads to a result by V.P. Saxena [5, p. 66, eq. (4.5.1)]:

$$\int_0^\infty x^{\eta-1}\, I_{p_i,q_i:r}^{m_1,n_1}\left[z x^{\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,n_1};(a_{ji},\alpha_{ji})_{n_1+1,p_i}\\ (b_j,\beta_j)_{1,m_1};(b_{ji},\beta_{ji})_{m_1+1,q_i}\end{matrix}\right.\right]$$

$$\times H_{p,q}^{m,n}\left[w x\left|\begin{matrix}(c_j,\gamma_j)_{1,n};(c_j,\gamma_j)_{n+1,p}\\ (d_j,\delta_j)_{1,m};(d_j,\delta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$= w^{-\eta}\, I_{p_i+q,q_i+p:r}^{m_1+n,n_1+m}\left[\; z w^{-\sigma}\left|\begin{matrix}(a_j,\alpha_j)_{1,n_1},\\ (b_j,\beta_j)_{1,m_1},\end{matrix}\right.\right.$$

$$(1-d_j-\eta\delta_j,\sigma\delta_j)_{1,m};(a_{ji},\alpha_{ji})_{n_1+1,p_i},$$

$$(1-c_j-\eta\gamma_j,\sigma\gamma_j)_{1,n};(b_{ji},\beta_{ji})_{m_1+1,q_i},$$

$$\left.\begin{matrix}(1-d_j-\eta\delta_j,\sigma\delta_j)_{m+1,q}\\ (1-c_j-\eta\gamma_j,\sigma\gamma_j)_{n+1,p}\end{matrix}\right]. \qquad (24)$$

On specializing the parameters of the I-function involved in (15), (17), (18), (19), (20) and (21), several other interesting new and known results may be obtained.

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MINTERMS GENERATIONS ALGORITHM USING WEIGHTED SUM METHOD

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ABSTRACT

The paper presents an exact algorithms for minterms generation. These algorithms are exact in the sense that they guarantee the minimum number of minterms terms in the final solution. Using this algorithm minterms may be generating from any minimized sum of product terms of multiple input variables. A flow chart is prepared to generate minterms called Minterms Generator. The Minterms Generator able to generate minterms from any number minimized sum of product terms of any number of input variables. This algorithm easily implement in computer programming. The completeness, consistency, and finite convergence of the algorithm are proven. Representative results from the computer program implementation of the algorithm are presented in this paper.

Keywords - Algorithm; minterms, minimization; Boolean function; weighted sum.

I. NTRODUCTION

The designs of digital systems are mainly determination of Boolean expression from the given information and to implement by using suitable gates. The K-map for n numbers of input variables consist of 2^n squares because n variables can be combined to from 2^n minterms [1]. As the number of variables increases, the excessive number of squares prevents a reasonable selection of adjacent squares. Disadvantages of the map are that it is essentially a trial and error procedure, which relies on the ability of the user to recognize certain patterns. It is very effective for the minimization of expression with up to 4 inputs variables and it complexity increase as no of input variable increase. It depends on the ability to visually identify prime applicants and select a set of prime implicants that cover all minterms [2]. This is not a direct algorithm to be implemented in a computer. An algebraic approach based primarily on successive expansion to generate all the prime implicants of a Boolean function utilizing the maxterm-type expression was first proposed by Nelson [3-4]. This basic idea of Nelson was subsequently utilized by Das and Choudhury in developing a tabular method for a more efficient generation of all the prime implicants of a Boolean function starting from the maxterm type expression represented in decimal mode [5]. For handling larger inputs a programmable method McCluskey proposed an algorithmic based technique for simplifying Boolean logic functions. These problems can be described by specifying the fundamental products to be included in the function where to minterms are combined together if there binary representation differs by a single bit replaced by (-). In addition, a modification of this basic technique has been developed which permits the direct generation of only the essential prime implicants [6]. Standard ways to represent any Boolean logics are "Sum of Products" (SOP).

II. PRELIMINARIES

In this section, present the basic notation and definitions used in the sequel. Some of the definitions below have been taken from [7].

Completely specified input variables defined as which take any values 0 or 1 for all the un-minimized input variables.

Incompletely specified input variables, which takes any values 0, 1 or the output may also take don't care values (–) for some of the un-minimized input variables.

Weight i.e. power of 2 depends on the position of the input variables. As for example Bi indicate the input variable at i-th position so weight of Bi is 2i.

Weighted sum is the addition of the weight values according their bit position of the variable.

P Matrix is a representation technique of SOP terms of minimized input variables.

Let, an n input combinational circuit represent by function $f(B_n, B_{(n-1)}, B_{(n-2)}, \ldots B_3, B_2, B_1)$.

After minimization, $f(B_n, B_{n-1}, B_{n-2}, \ldots B_3, B_2, B_1) = B_{n-1}\bar{B}_2 B_1 + B_n \bar{B}_{n-1} B_i \bar{B}_2 \bar{B}_1$.

Completely specified variable inputs are $B_{(n-1)}, B_2$ and B_1 because in the both SOP terms they are always either 0 or 1 no don't care values (–). Rests of the variables inputs are incompletely specified. P matrix representation of the above facts,

$$P = \begin{bmatrix} B_n & B_{n-1} & B_{n-2} & \cdot & \cdot & B_i & \cdot & \cdot & B_3 & B_2 & B_1 \\ - & 1 & - & - & - & - & - & - & - & 0 & 1 \\ 1 & 0 & - & - & - & 1 & - & - & - & 0 & 0 \end{bmatrix}$$

II. RULE FOR MINTERMS GENERATIONS

In general, if numbers of un-minimized input variable are n, after minimization input variable are m, then the numbers of minterms required for minimization from n input variable to m input variable are $2^{(n-m)}$ because (n-m) inputs must be take all possible combination of binary value. Let an e.g. of P_1 matrix, here x indicate either 0 or 1 fixed value so, B_n, B_2 and B_1 these three inputs are completely specified variable inputs, rest (n-3) variables take both the combination of 0 and 1.

$$P_1 = \begin{bmatrix} B_n & B_{n-1} & B_{n-2} & \cdot & B_i & \cdot & \cdot & B_3 & B_2 & B_1 \\ x_n & - & - & - & - & - & - & - & x_2 & x_1 \end{bmatrix}$$

So, possible combinations of incompletely specified input variables are $2^{(n-3)}$ and their weighted sum generate minterms. Let, m_i represent minterms. Where, r = 0,1,2, … $2^{(n-m)}$.

III. MINTERMS GENERATOR

A Flowchart for Minterms Generation, called it Minterms Generator (Fig 1).

Here, m_0 is lowest weighted value of minterms and W positional weight of the incompletely specified input variables in increasing way.

Let a 8 input system describe by SOP form *f(A,B,C,D,E,F,G,H)=A**B'** CD+**A'** DEG'+B**F'** **G**H.*

P matrix representation as below,

$$P = \begin{bmatrix} A\,B\,C\,D\,E\,F\,G\,H \\ 1\;0\;1\;1\;-\;-\;-\;- \\ 0\;-\;-\;1\;1\;-\;0\;- \\ -\;1\;-\;-\;-\;0\;1\;1 \end{bmatrix}$$

For 1st row constant inputs are A, B, C and D. Lowest significant variable input is H. Here $m_{1,0}$=128+0+32+16=176.

Next Minterms generate by changing the value of H from 0 to 1 keeping all other higher incompletely specified input are 0. Weighted value of H is $W = 2^0 = 1$. Minterms increase up to 1st position $m_{1,1}$=1+176=177. For G = 1, all higher incompletely specified input are 0 and H can take two vales 0 and 1. Since only H is lower variable are 2 so only two minterms generate m_2 and m_3. Weighted value of G is $W = 2^1 = 2$. TABLE I represent the generate minterms keeping all variable except H.

For F, higher incompletely specified input is 0. Lower incompletely specified variable inputs G and H can take all possible combination of 0 and 1. Since possible combination are 2^n where n = no of lower variable input i.e. 4. So, only 84minterms are generates from $m_{1,4}$ to $m_{1,7}$. Weighted value of F is $W = 2^2 = 4$. TABLE II represent the generate minterms keeping all variable except G and H.

For E, no higher incompletely specified variable inputs present. Lower incompletely specified variable inputs F, G and H can take all possible combination of 0 and 1. Since possible combination are 2^3=8. So, only 8 minterms are generates from $m_{1,8}$ to $m_{1,15}$. Weighted value of E is $W = 2^3 = 8$. TABLE V represent the generate minterms keeping all variable except F, G and H.

TABLE III represent the generate minterms keeping all variable except F, G and H.

So, generated minterms for 1st row are 176,177,178, 179,18 0,181,182,183,184,185,186,187,188,189,190 and 191.

For 2nd row constant inputs are A, D, E and G. Lowest significant variable input is H. Here $m_{2,0}$=0+16+8+0=24. Next Minterms generate by changing the value of H from 0 to 1 keeping all other higher variable inputs are 0. Weighted value of H is $W = 2^0 = 1$. Minterms increase up to 1 position $m_{2,1}$=1+24=25.

For F = 1, all higher variable inputs are 0 and H can take two vales 0 and 1. Since only H is lower variable input and possible combination are 2 so only two minterms generate $m_{2,2}$ and $m_{2,3}$. Weighted value of G is $W = 2^2 = 4$. TABLE IV represent the generate minterms keeping all variable except H for 2nd row of inputs.

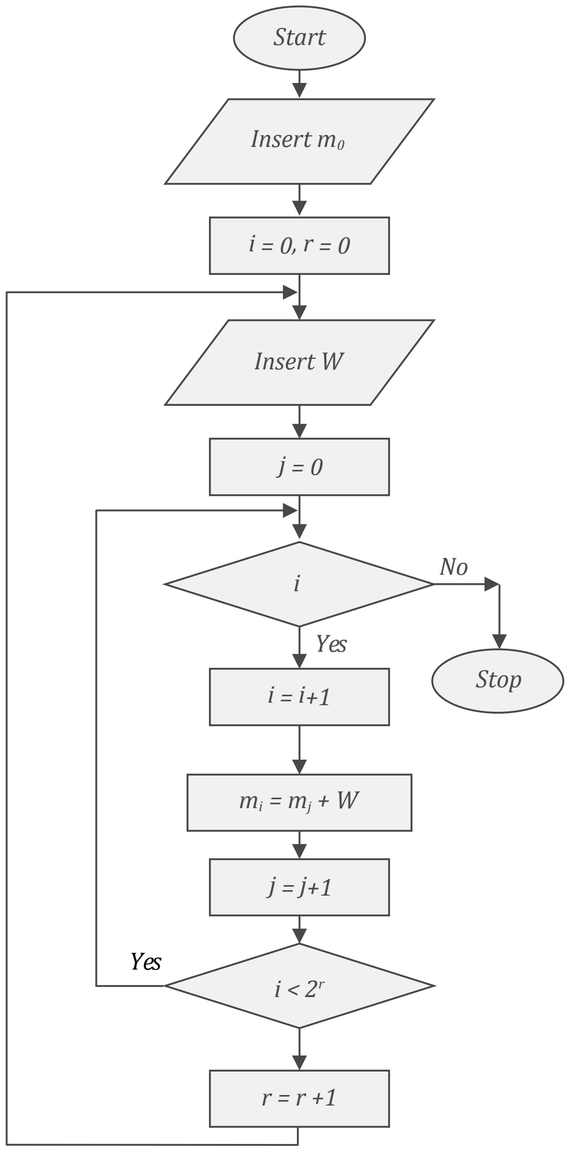

Fig-1: Flow chart for generate minterms.

Table I. Minterm Generation Table
By Keeping H Variable

Value of Variable Inputs		Generated Minterms
H	Decimal Value	
0	0	$m_{1,2}$= 2+ $m_{1,0}$=178
1	1	$m_{1,3}$= 2+ $m_{1,1}$=179

Table II. Minterm Generation Table
By Keeping G And H Variable

Value of Variable Inputs			Generated Minterms
G	H	Decimal Value	
0	0	0	$m_{1,4}$= 4+ $m_{1,0}$=180
0	1	1	m1,5= 4+ $m_{1,1}$=181
1	0	2	$m_{1,6}$= 4+ $m_{1,2}$=182
1	1	3	$m_{1,7}$= 4+ $m_{1,3}$=183

Table III. Minterm Generation Table
By Keeping F,G And H Variable

Value of Variable Inputs				Generated Minterms
F	G	H	Decimal Value	
0	0	0	0	$m_{1,8}$=8+ $m_{1,0}$=184
0	0	1	1	$m_{1,9}$=8+ $m_{1,1}$=185
0	1	0	2	$m_{1,10}$=8+ $m_{1,2}$=186
0	1	1	3	$m_{1,11}$= 8+ $m_{1,3}$=187
1	0	0	4	$m_{1,12}$= 8+ $m_{1,4}$=188
1	0	1	5	$m_{1,13}$= 8+ $m_{1,5}$=189
1	1	0	6	$m_{1,14}$= 8+ $m_{1,6}$=190
1	1	1	7	$m_{1,15}$= 8+ $m_{1,7}$=191

Table IV. Minterm Generation Table
By Keeping H Variable For 2Nd Row Of Input

Value of Variable Inputs		Generated Minterms
H	Decimal Value	
0	0	$m_{2,2}$= 4+ $m_{2,0}$=28
1	1	$m_{2,3}$= 4+ $m_{2,1}$=29

Table V. Minterm Generation Table
By Keeping F And H Variable

Value of Variable Inputs			Generated Minterms
F	H	Decimal Value	
0	0	0	$m_{2,4}$= 32+ $m_{2,0}$=56
0	1	1	$m_{2,5}$= 32+ $m_{2,1}$=57
1	0	2	$m_{2,6}$= 32+ $m_{2,2}$=60
1	1	3	$m_{2,7}$= 32+ $m_{2,3}$=61

Table VI. Minterm Generation Table
By Keeping C,F And H Variable For 2nd Row Of Input

Value of Variable Inputs				Generated Minterms
C	F	H	Decimal Value	
0	0	0	0	$m_{2,8}$= 64+ $m_{2,0}$=88
0	0	1	1	$m_{2,9}$= 64+ $m_{2,1}$=89
0	1	0	2	$m_{2,10}$= 64+ $m_{2,2}$=92
0	1	1	3	$m_{2,11}$= 64+ $m_{2,3}$=93

1	0	0	4	$m_{2,12}$= 64+ $m_{2,4}$=120
1	0	1	5	$m_{2,13}$= 64+ $m_{2,5}$=121
1	1	0	6	$m_{2,14}$= 64+ $m_{2,6}$=124
1	1	1	7	$m_{2,15}$= 64+ $m_{2,7}$=125

For C=1, higher variable inputs is 0. Lower variable inputs F and H can take all possible combination of 0 and 1. Since possible combination are 2^n where n = no of lower variable input i.e. 4. So, only 4 minterms are generates from $m_{2,4}$ to $m_{2,7}$. Weighted value of C is ***W= 2⁶=32***. TABLE V represent the generate minterms keeping all variable except F and H.

For B= 1, no higher variable inputs present. Lower variable inputs C, F and H can take all possible combination of 0 and 1. Since possible combination are 2^3=8.So, only 8 minterms are generates from $m_{2,8}$ to $m_{2,15}$. Weighted value of B is ***W= 2^7=64***. TABLE VIII represent the generate minterms keeping all variable except C, F and H. TABLE VI represent the generate minterms keeping all variable except F and H.

So, generated minterms for 2nd row are 24,25,29,56,57,60,61,88,89,92,93, 120, 121,124 and 125.

For 3rd row constant inputs are A, C, D and E.
For 3rd row constant inputs are B, F, G and H. Here $m_{3,0}$=64+0+2+1=67. Next Minterms generate by changing the value of E from 0 to 1 keeping all other higher variable inputs are 0. Weighted value of E is ***W= 2^3=8***. Minterms increase up to 1 position $m_{3,1}$=8+67=75. For D = 1, all higher variable inputs are 0 and E can take two vales 0 and 1. Since only E is lower variable input and possible combination are 2 so only two minterms generate $m_{3,2}$ and $m_{3,3}$. Weighted value of G is ***W= 2^4=16***. TABLE VII represents the generated minterms keeping all variable except E.

For C=1, higher variable inputs is 0. Lower variable inputs F and H can take all possible combination of 0 and 1. Since possible combination are 2^n where n = no of lower variable input i.e. 4. So, only 4 minterms are generates from m_4 to m_7. Weighted value of C is ***W= 2^5=3***2. TABLE VIII represents the generated minterms keeping all variable except D and E.

Table VII. Minterm Generation Table
By Keeping E Variable

Value of Variable Inputs		Generated Minterms
E	Decimal Value	
0	0	$m_{3,2}$= 16+ $m_{3,0}$=83
1	1	$m_{3,3}$=16+ $m_{3,1}$=91

Table VIII. Minterm Generation Table
By Keeping D And E Variable

Value of Variable Inputs			Generated Minterms
D	E	Decimal Value	
0	0	0	$m_{3,4}$= 32+ $m_{3,0}$=99
0	1	1	$m_{3,5}$= 32+ $m_{3,1}$=107
1	0	2	$m_{3,6}$= 32+ $m_{3,2}$=115
1	1	3	$m_{3,7}$= 32+ $m_{3,3}$=123

Table IX. Minterm Generation Table
By Keeping C,D And E Variable

Value of Variable Inputs				Generated Minterms
C	D	E	Decimal Value	
0	0	0	0	$m_{3,8}$= 128+ $m_{3,0}$= 195
0	0	1	1	$m_{3,9}$= 128+ $m_{3,1}$= 203
0	1	0	2	$m_{3,10}$=128+ $m_{3,2}$= 211
0	1	1	3	$m_{3,11}$=128+ $m_{3,3}$=219
1	0	0	4	$m_{3,12}$= 128+ $m_{3,4}$=227
1	0	1	5	$m_{3,13}$= 128+ $m_{3,5}$=235
1	1	0	6	$m_{3,14}$= 128+ $m_{3,6}$=243
1	1	1	7	$m_{3,15}$= 128+ $m_{3,7}$=251

For A= 1, no higher variable inputs present. Lower variable inputs C, F and H can take all possible combination of 0 and 1.
Since possible combination are 2^3=8. So, only 8 minterms are generates from m_8 to m_{15}. Weighted value of A is ***W= 2^7=128***. TABLE IX represents the generated minterms keeping all variable except C, D and E.

So, generated minterms for 3rd row are 67,75,83,91,99,107, 115,123,195,203,211,219,227,235,243 and 251.

So, generated minterms for the P matrix are given below, <u>24</u>, <u>25</u>, <u>28</u>, <u>29</u>, <u>56</u>, <u>57</u>, <u>60</u>, <u>61</u>, 67,75, 83, <u>88</u>, <u>89</u>, 91, <u>92</u>, <u>93</u>, <u>99</u>,107,115, <u>120</u>, <u>121</u>, 123, <u>124</u>, <u>125</u>, ***176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191***,195,203,211,219, 227,235, 243,251.
Bold and Italic terms are for row 1 under lines terms are generated minterms for row 2, and plain text are for row 3 for the P matrix. Reversely we say that minimization expression is *f(A,B,C,D,E,F,G,H)=A**B'** CD+**A'** DE**G'**+B**F'** **G**H.*
By using this rule minterms may be generate which may be use in extended and modified tabular formula [8-12].

IV. CONCLUSION

This paper studied the condition of minimizing Boolean expressions and proposed an optimal method of Boolean function simplification using weighted sum. It can be effectively and easily implemented for problems having large numbers of input variables. Further research should be conducted to develop an algorithm that uses this new method. The proposed method may lead to easy ULSI fabrication and finally an attempt to use this technique on Extra Large Scale Integrations (ELSI).

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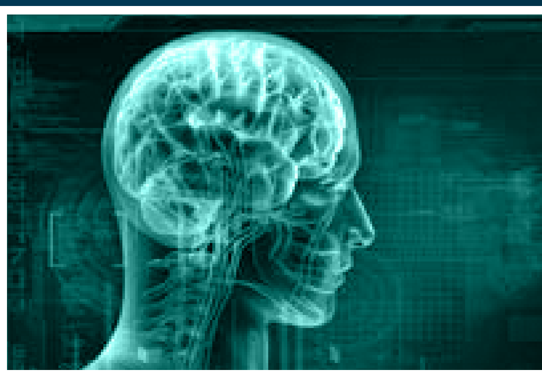
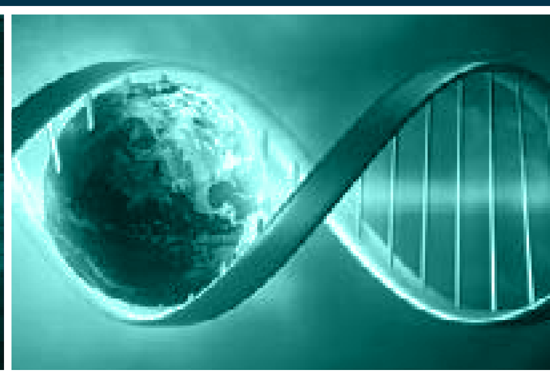

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